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Đuro Kurepa memorial volume

ĐURO KUREPA
(1907 – 1993)

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At the moment of the appearance of this article it will be two years since Professor Đuro Kurepa, the great Yugoslav mathematician, died. The aim of this paper is to give a short review of his scientific biography.

Đuro Kurepa was born on August 16, 1907 in Majske Poljane near Glina in Srpska Krajina as the fourteenth and last child of Rade and Anđelija Kurepa. He went to elementary and high school in Majske Poljane, Glina and Križevci. Then, he got his diploma in theoretical mathematics and physics at the Faculty of Philosophy of the University of Zagreb in 1931. Kurepa spent the years 1932–1935 in Paris at the Faculté des Sciences and the Collège de France. There he obtained his doctoral diploma at the Sorbonne in 1935 before a committee whose members were Paul Montel (president), Maurice Fréchet (supervisor), and Arnaud Denjoy. He received his post-doctoral education at some of the world's best institutions: the University of Warsaw and the University of Paris (1937), and after the Second World War he visited Cambridge (Massachusetts), the mathematical departments of the Universities of Chicago, Berkeley and Los Angeles, and the Institute of Advanced Studies in Princeton.

Kurepa's first employment was at the University of Zagreb in 1931, as an assistant in mathematics. He became an assistant professor at the same institution in 1937, associate professor in 1938, and full professor in 1948. He stayed in Zagreb until 1965 when he moved to Belgrade where he was invited to be full professor at the Faculty of Science of the University of Belgrade. He stayed there until his retirement in 1977. Meanwhile, he was a visiting professor at Columbia University in New York (Summer School 1959), and Boulder, Colorado, in 1960. Besides his university teaching, Kurepa organized successfully scientific work, as well, and was very active in administrative matters. Professor Kurepa was the chairman of the Mathematical Department of the Faculty of Philosophy in Zagreb; then since 1970

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till 1980 chairman of the Mathematical Seminar of the Institute of Mathematics of SANU (the Serbian Academy of Sciences and Arts). He was a full member of SANU, the Academy of Science of Bosnia and Herzegovina, and a corresponding member of JAZU (Yugoslav Academy of Sciences and Arts) in Zagreb.

Professor Kurepa was the founder and president of the Society of Mathematicians and Physicists of Croatia, and president of the Union of Yugoslav Societies of Mathematicians, Physicists and Astronomers. He was also president of the Yugoslav National Committee for Mathematics, as well as president of the Balkan Mathematical Society. Furthermore, he was the founder and for many years the chief editor of the journal *Mathematica Balkanica*, now published in Sofia. Kurepa was also a member of the editorial board of Belgrade's mathematical journals *Publications de l'Institut Mathématique*, *Vesnik* and the German journal *Zeitschrift für mathematische Logik und Grundlagen der Mathematik*.

Professor Kurepa earned many awards, honors and distinctions. He received the highest prize of former Yugoslavia, the Award of AVNOJ (1976). Also, he was a member of the Tesla Memorial Society of the U.S.A. and Canada (1982), the Bernhard Bolzano Charter, and the Gramata Marin Drinov of the Bulgarian Academy of Science (Sofia 1987).

The scientific output of Đuro Kurepa was rather large. He published about 200 scientific papers, and more than 700 writings: books, articles, reviews. His papers were published in journals all around the world, and some of them in the most recognized mathematical journals such as: *Matematische Annalen, Izvestiya Akademii nauk SSSR, Fundamenta Mathematicae, Acta Mathematica, Comptes Rendus de l'Académie des sciences, Bulletin de la Société Mathématique de France, Zeitschrift für mathematische Logik und Grundlagen der Mathematik, Journal of Symbolic Logic, Pacific Journal of Mathematics.* Many of his manuscripts were translated into English, French, Italian, and other languages. Kurepa also wrote about 1000 scientific reviews. He delivered lectures at many universities of Europe, America and Asia; for example, at Warsaw, Paris, Moscow, Jerusalem, Istanbul, Cambridge, Boston, Chicago, Berkeley, Princeton and Peking. As Kurepa himself told once: "I lectured at almost each of nineteen universities of (former) Yugoslavia, then in almost every European country, then in Canada, Cuba, Israel and Iraq, and I gave at least ten lectures in each of the following countries: France, Italy, Germany, the Soviet Union and the United States." He participated at dozens of international mathematical symposia, and many of them were organized by himself (for example, the international conferences in topology in Herceg Novi 1968, Budva-Bečići 1972, Belgrade, 1977.).

The influence of Professor Kurepa on the development of mathematics in Yugoslavia was great. As a professor of the University of Zagreb he introduced several mathematical disciplines, mainly concerning the foundations of mathematics and set theory. This is best witnessed by the following words of Kajetan Šeper, a professor of the University of Zagreb:

"Professor Kurepa was not only the professional mathematician and teacher, but he was a scientist, philosopher and humanist as well, in the true sense of these words. He was the founder and pioneer in mathematical logic and the foundations of mathematics in Croatia, and modern mathematical theories in Croatia and Yugoslavia. Generally speaking, he was the catalyzer, the initiator and the bearer of mathematical science."[1]

His arrival to Belgrade in the mid-sixties, and the subsequent influence he had on the mathematical community there, may be described with almost the same words. Professor Kurepa exposed the newest results in diverse mathematical disciplines through many seminars, courses and talks which he delivered at the Faculty of Sciences and the Mathematical Institute. The topics of his lectures included: the construction of Cohen forcing, some questions concerning independence results in cardinal and ordinal arithmetic, ordered sets and general topology. But he was attracted to other mathematical topics, too. He gave valuable contributions to analysis, algebra, number theory, and even to those mathematical disciplines which were just appearing, as computer science, for example. The universality of his spirit is portrayed by the list of university courses he taught: Algebra, Analysis and Topology and Set Theory.

By publishing his doctoral dissertation *in extenso* in *Publications Mathématiques de l'Université de Belgrade*, **4** (1935), 1–138, Kurepa made a first contact with the Belgrade mathematical community. In the beginning of the fifties these contacts became deeper and more frequent. So, from a text[2] of Professor Zlatko Mamuzić we learn that Kurepa was invited already in 1952 to visit the University of Belgrade. On this occasion he gave talks on the theory of matrices, and held a seminar with topics in set theory, topology and algebra. In the same text we find the names of attendants of the seminar: Professor Mamuzić himself, Časlav Stanojević (who later became Professor of the University of Rolla, Missouri), Simon Ćetković (later Professor of the Faculty of Electric Engineering in Belgrade), Mirko Stojaković (Professor of Algebra of the University of Novi Sad) and others. By attending these seminars, many mathematicians gained ideas for their mathematical papers, while graduate students obtained themes for their master and doctoral theses. Many of these themes were formulated or initiated by Kurepa. These works include virtually all doctoral theses of the older generation of topologists, and many algebraists from all over Yugoslavia: Svetozar Kurepa, Zlatko Mamuzić, Sibe Mardešić, Pavle Papić, Viktor Sedmak, and a few years later, Ljubomir Ćirić, Rade Dacić, Milosav Marjanović, Veselin Perić, Milan Popadić, Ernest Stipanić and Mirko Stojaković. Professor Kurepa was supervisor (altogether 42 times) or a member of examination boards for doctoral dissertations for many other mathematicians: Miroslav Ašić, Nataša Božović, Dušan Ćirić, Aleksandar Ivić, Milena Jelić, Gojko Kalajdžić, Ljubiša Kočinac, Žarko Mijajlović, Nada Miličić, Mila Mršević, Marica Prešić, Zoran Šami, Stevo Todorčević, Ratko Tošić, and others. Many of these mathematicians continued and developed further Kurepa's work, notably Stevo Todorčević.

[1] Set theory. Foundations of mathematics. Symposium dedicated to Đuro Kurepa. Belgrade 1977

[2] *Povodom 35 godina knjige ,,Teorija skupova" profesora Djure Kurepe* (*On the occasion of 35 years of the book "Set theory" of Đuro Kurepa*), Istorijski spisi iz matematike i mehanike, Istorija matematičkih i mehaničkih nauka, knj. 2, Matematički institut, Beograd, 1989.

Professor Kurepa had contacts with many mathematicians of the highest rank from all around the world. Thanks to him some of them visited Belgrade: A. Tarski, P. Alexandroff, M. Krasner, N.A. Shanin, K. Devlin and others. Professor Kurepa especially was proud of his encounter with Nikola Tesla, the great Serbian scientist and engineer, with whom Kurepa was fascinated.

Now we shall describe shortly Kurepa's work in topology, set theory and number theory.

In topology Kurepa was interested in some generalizations (non-numerical) of distance functions. In this context there is a notion of Kurepa's pseudo-metric spaces. As it was already mentioned, in the middle of thirties Kurepa was a doctoral student in Paris, and at that time he was influenced by the French mathematical school, particularly by the work of M. Fréchet. Along these lines, Kurepa took a new approach to the notion of space. He defined the notion of pseudo-distancial space (espaces pseudo-distanciés)[3] generalizing the Fréchet's class (D). In this approach, values of distance function range over a totally ordered set, instead of the set of positive reals, and the triangle condition on distance function is replaced by an interesting relation in ordered sets. Later Frechet arrived at the same notion[4], and since then this class of abstract spaces are known as "Kurepa-Fréchet spaces". A lot of results on this classes of spaces are obtained by Z. Mamuzić, P. Papić, A. Appert, J. Colmez, V.G. Boltjanski and others. It is interesting that Kurepa wrote last time about these spaces in 1992[5]. Someone probably would recognize in this class of spaces the notion of Zadeh's fuzzy sets. Anyway, there is the phrase "sets of fuzzy structure"[6] in his book *Set theory* from 1951.

Trees, partially ordered sets in which every lower cone is a well-ordered set, may be considered as a natural generalization of ordinal numbers. They are a special type of *ramified sets* which Kurepa introduced in his doctoral thesis "Ensembles ordonnés at ramifiés"[7]. By widespread opinion[8], this capital work is a first systematic study on set-theoretical trees. In his thesis, and later in his papers "Ensembles lineaires et une classe de tableaux ramifiés"[9] and "A propos d'une generalisation de la notion d'ensamles bien ordonnés"[10] Kurepa introduced fundamental notions of the theory of infinite trees: Aronszajn tree (this is an infinite tree of the cardinality and height $\aleph_1$ in which every chain and every level is at most countable), Suslin tree (this is an Aronszajn tree in which every antichain is countable), and Kurepa tree (this is a tree of the cardinality and height $\aleph_1$ which has at least $\aleph_2$ branches of the

[3] *Tableaux ramifiés d'ensembles, Espaces pseudo-distancies*, C.R. 1938, Paris (1934), 1563-1565.

[4] It is interesting that Frechete was not aware of Kurepa's work, but during his visit to Belgrade after the Second World War he was acquainted with it.

[5] *General Ecart*, Simp. Filomat '92, Niš October 8-10, Zb. radova Fil. fak. u Nišu, Ser. mat. 6;2(1992), 373-379

[6] *skupovi zamazane stukture*

[7] Kurepa defended his doctoral thesis in Paris in 1935. This thesis is published in whole in Publ. Math. Univ. Belgrade, **4**, 1–148

[8] see, for example K. Kunen, *Set Theory,* North-Holland, Amsterdam, 1983, p. 69

[9] Publ. Math. Univ. Belgrade, **6**, 129–160.

[10] Acta Mathematica **75** (1942), 139–150.

height $\aleph_1$). Thus, Kurepa found in Suslin and Kurepa trees two extremal notions, the first one without long chains (i.e. of the length $\aleph_1$), while the Kurepa tree has a lot of them, at least $\aleph_2$. Kurepa proved many interesting properties concerning these objects. Probably the best known is the following equivalence with the Suslin hypothesis

$$\text{SH} \Leftrightarrow \textit{There is no Suslin tree.}$$

Here, SH denotes notable Suslin hypothesis that there is no Suslin line, i.e. a linearly ordered set $\mathcal{L}$ of the countable celularity (i.e. every set of pairwise disjoint intervals of $\mathcal{L}$ is at most countable), but which does not have a countable dense subset.

Lebesgue in his paper in 1905 identified implicitly analytic functions with Bair functions. In his proof he used an argument which was "simple and short, but wrong". The mistaken step in the proof was hidden in the proof of a lemma which he considered trivial, namely that a projection of a Borel set is also a Borel set. Ten years later, Suslin, a young and talented Luzin's student found the mistake. Suslin introduced the notion of analytic set, as a projections of Borel sets, and he proved that there are analytic sets that are not Borel sets. So emerged descriptive set theory, one of the deepest and most interesting parts of set theory. However, Suslin died soon (in 1919), and the formulation of Suslin hypothesis appear after his death in his paper "Probleme 3"[11] a year later. This hypothesis will play the central role in the development of the theory of infinite trees, and in this progress Kurepa's work was of the principal importance. Namely, Kurepa was trying since 1935 to solve SH. He did not succeed in this, simply it was not possible to solve it in this time. Tools of the classical set theory that Cantor founded, and Zermelo, Fraenkel, Hausdorff, König, Tarski and others developed, were not adequate. However, Kurepa was the first who understood the importance of trees in set theory. Some set-theorists discovered later again properties of these partially ordered sets. For example, Miller in 1943, and Sierpinski in 1948 independently discovered the mentioned equivalence to Suslin hypothesis which Kurepa found already in 1935. Using infinite trees, Kurepa found examples of topological spaces with important and unusual properties. One example of this kind is connected with Suslin line. Namely, Kurepa proved[12] that the topological square of the Suslin continuum $\mathcal{K}$ has uncountable celularity, while $\mathcal{K}$ itself has countable celularity.

Kurepa was not able to prove the existence or nonexistence of Suslin tree, neither of Kurepa tree. The postulate that there is a Kurepa tree was named Kurepa hypothesis, shortly KH. The complete solution of the problem of the existence of these trees was solved in the beginning of seventies, when the new method, Cohen's forcing, became a standard and prime tool in set theory. So Solovay, Tennenbaum and Jensen proved that SH is independent from ZFC+CH (Zermelo–Fraenkel set theory plus Continuum Hypothesis), while Devlin proved in 1978 that

[11] Fund. Math., **1**, 223

[12] *La condition de Suslin et une propriètè charactéristique des nombres réels*, C.R. Acad. Sci. Paris **231**, (1950), 1113–1114

every theory[13]

$$\mathrm{ZFC} \pm \mathrm{CH} \pm \mathrm{SH} \pm \mathrm{KH}$$

is consistent.[14] This fact shows that the nature of the postulates SH and KH differs from the others axioms of ZFC. Hence, it is clear why these structures play so important role in various constructions in set theory, general topology, model theory and infinite combinatorics.

Kurepa had a distinguished ability to sense a good problem and a fine construction, especially if they are connected to ordered sets. We cannot mention all examples of this kind, but one problem from number theory deserves a special attention, as it was considered by several Yugoslav mathematicians and mathematicians from abroad. Kurepa formulated during a mathematical gathering in Ohrid in 1971 the following problem. First he defined an arithmetical function $!n$ which he called "the left factorial function" as a sum of factorials of first $n-1$ non-negative integers. Thus,

$$!n = 0! + 1! + 2! + \ldots + (n-1)!.$$

Then the formulation of $!n$-hypothesis is stated as follows: The greatest common divisor of $!n$ and $n!$ is 2. This hypothesis has a lot of interesting equivalences, and it was considered by L. Carlitz, Wagstaff, W. Keller, and Yugoslav mathematicians Slavić, Šami, Žižović, Stanković, Gogić, Ivić, Mijajlović, and others. This hypothesis is stated in R. Guy's book *Unsolved problems in number theory,* Springer-Verlag, 1981, as problem B44. The hypothesis was tested by computers for $n < 1000000$ (Mijajlović, Gogić in 1991). Kurepa announced the solution (he rang me up one early morning in the spring 1992 to tell me this), but he never published the solution. R. Guy in a letter to me in 1991 mentioned that R. Bond from Great Britain might solved the left factorial hypothesis, but this proof did not appear either up to now.

Kurepa was attracted by many areas of mathematics, besides set theory, general topology, foundations, and number theory. His work includes also topics in algebra (theory of matrices), numerical mathematics, computer science and fixed-point theory. It is not possible to discuss here his full mathematical achievements. Let us mention that several paperss on this matter are already published. There is also some further discussion in introductory texts of the book *Selected Works of Đuro Kurepa* (eds.: S. Todorčević, Z. Mamuzić, A. Ivić and Ž. Mijajlović) which is in preparation in the Mathematical Institute, Belgrade. In short we may say: Đuro Kurepa has great merits for the development of the foundations of set theory and mathematics in general.

[13] for details see Todorčević's article *Trees and linearly ordered sets* in: K. Kunen and J.E. Vaughan (eds), Set-Theoretic Topology, North-Holand, Amsterdam, 1985, pp. 235–294

[14] here, for a sentence φ, $\pm\varphi$ denotes either φ, or the negation of φ