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RELATIVE COMPLEMENTS IN THE WEAK CONGRUENCE LATTICE

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Abstract. Necessary and sufficient conditions under which a weak congruence lattice of an algebra is relatively complemented are given. Consequences concerning other kinds of complementedness are also presented. Finally, we investigate which of the main algebraic constructions preserve the relative complements in the weak congruence lattice.

1. Introduction

Weak congruences have been extensively investigated over the last decade. The lattice of these relations on an algebra turns out to be a useful tool for understanding connections among congruences of subalgebras and subalgebras themselves. Thus it is possible to characterize algebras and varieties by properties of weak congruence lattices (e.g., [**1, 9, 11, 8**]). Another direction in these investigations are descriptions of weak congruence lattices for particular classes of algebras (e.g., [**5, 6, 7**]). Finally, problems of representations of algebraic lattices by weak congruences were also discussed (e.g., [**2, 10**]). Generalizations of weak congruences by a wider class of compatible relations were given in [**3**].

The aim of the present paper is to characterize algebras having relatively complemented lattices of weak congruences. We give necessary and sufficient conditions under which an algebra satisfies this property. In addition, complementedness of this lattice is investigated, and improvements of the results from [**9**] are obtained.

2. Preliminaries

We advance some basic notions concerning weak congruences, and also some relevant lattice-theoretic properties of special elements in a lattice. For more details and other properties of weak congruences lattices, we refer to the list of papers, given in References.

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A *weak congruence* ρ of an algebra $\mathcal{A} = (A, F)$ is a symmetric and transitive relation on A, which is also compatible with all fundamental operations (where compatibility with nullary operations is also assumed—for a nullary operation c, $c\,\rho\,c$ is satisfied). Obviously, a nonempty weak congruence on $\mathcal{A}$ is an ordinary congruence on a subalgebra of $\mathcal{A}$. By the definition, $\emptyset$ is also a weak congruence if and only if there are no nullary operations in F. A set of all weak congruences of an algebra $\mathcal{A}$ is an algebraic lattice under inclusion and is denoted by $\mathrm{Cw}\,\mathcal{A}$ [**11**].

We denote the diagonal relation of a set A by Δ_A, or simply by Δ when there is no danger of confusion. The diagonal relation Δ is always a codistributive element in a weak congruence lattice, i.e., for all $\rho, \theta \in \mathrm{Cw}\,\mathcal{A}$, the following holds:

$$\Delta \wedge (\rho \vee \theta) = (\Delta \wedge \rho) \vee (\Delta \wedge \theta).$$

Obviously, the filter $\uparrow\!\Delta$ is the congruence lattice $\mathrm{Con}\,\mathcal{A}$. The ideal $\downarrow\!\Delta$ is isomorphic with $\mathrm{Sub}\,\mathcal{A}$ (the lattice of subuniverses of $\mathcal{A}$).

An algebra $\mathcal{A}$ satisfies the *Congruence Extension Property* (the CEP) if every congruence of any subalgebra of $\mathcal{A}$ is a restriction of a congruence of $\mathcal{A}$. An algebra $\mathcal{A}$ satisfies the *Congruence Intersection Property* (the CIP) if for each $\rho \in \mathrm{Con}\,\mathcal{B}$, $\theta \in \mathrm{Con}\,\mathcal{C}$, $\mathcal{B}, \mathcal{C} \in \mathrm{Sub}\,\mathcal{A}$, the following holds:

$$(\rho \cap \theta)_{\mathcal{A}} = \rho_{\mathcal{A}} \cap \theta_{\mathcal{A}},$$

where $\rho_{\mathcal{A}}$ is the smallest congruence of $\mathcal{A}$ containing ρ. In particular, if $\theta \in \mathrm{Con}\,\mathcal{A}$, then the above equality becomes

$$(\rho \cap \theta)_{\mathcal{A}} = \rho_{\mathcal{A}} \cap \theta,$$

and we say that an algebra $\mathcal{A}$ satisfying this condition has the *weak* CIP (the wCIP). An algebra $\mathcal{A}$ satisfies the CEP if and only if the diagonal relation Δ is a cancellable element in $\mathrm{Cw}\,\mathcal{A}$, i.e., if and only if for all $\rho, \theta \in \mathrm{Cw}\,\mathcal{A}$,

$$\rho \wedge \Delta = \theta \wedge \Delta \text{ and } \rho \vee \Delta = \theta \vee \Delta \quad \text{imply} \quad \rho = \theta.$$

An algebra $\mathcal{A}$ satisfies the CIP if and only if Δ is a distributive element in $\mathrm{Cw}\,\mathcal{A}$, that is if and only if for all $\rho, \theta \in \mathrm{Cw}\,\mathcal{A}$, the following equality holds:

$$\Delta \vee (\rho \wedge \theta) = (\Delta \vee \rho) \wedge (\Delta \vee \theta).$$

Consequently, an algebra $\mathcal{A}$ has the wCIP if and only if for every congruence ρ of a subalgebra of $\mathcal{A}$ and a congruence θ of $\mathcal{A}$, the following is satisfied:

$$\Delta \vee (\rho \wedge \theta) = (\Delta \vee \rho) \wedge \theta.$$

If an algebra $\mathcal{A}$ satisfies the wCIP and the CEP, then for every subalgebra $\mathcal{B}$ of $\mathcal{A}$, $\mathrm{Con}\,\mathcal{B}$ is isomorphic to the interval $[\Delta, B^2 \vee \Delta]$ in $\mathrm{Cw}\,\mathcal{A}$, under $\rho \mapsto \rho \vee \Delta$.

In addition to distributive, codistributive and cancellable elements, the following lattice-theoretic notions are also used throughout the paper (see e.g., [**4**]). An element a in a lattice L is *neutral* if for all $x, y \in L$, the following equality holds:

$$(x \vee y) \wedge (y \vee a) \wedge (a \vee x) = (x \wedge y) \vee (y \wedge a) \vee (a \wedge x).$$

An element a in a lattice L is *standard* if for all $x, y \in L$, we have that

$$x \wedge (a \vee y) = (x \wedge a) \vee (x \wedge y).$$

An element a is a neutral element of a lattice L if and only if it is distributive, codistributive and cancellable if and only if it is standard and codistributive. It follows that an algebra $\mathcal{A}$ satisfies CEP and CIP if and only if Δ is a neutral element in $\operatorname{Cw}\mathcal{A}$.

The *center* of a bounded lattice L is a set of all neutral elements from L which have complements. If L is a bounded lattice, and $a \in L$ belongs to its center, then

$$L \cong {\uparrow}a \times {\downarrow}a, \text{ under } x \mapsto (x \vee a, x \wedge a).$$

3. Results

Main properties of the weak congruence lattice of an algebra usually depend on its diagonal relation. Therefore, we begin with algebraic conditions under which there is a complement of the diagonal relation Δ in this lattice.

PROPOSITION 1. *The diagonal relation Δ has a complement in the weak congruence lattice $\operatorname{Cw}\mathcal{A}$ of an algebra $\mathcal{A}$ if and only if $\mathcal{A}$ has at least one nullary operation and no congruence of $\mathcal{A}$ has a block which is a proper subalgebra of $\mathcal{A}$.*

PROOF. Observe that the smallest subuniverse of $\mathcal{A}$ is not empty, since there are constants in $\mathcal{A}$. If Δ has a complement in the lattice $\operatorname{Cw}\mathcal{A}$, then the smallest subuniverse B_m of $\mathcal{A}$ has more than one element. Indeed, every complement Δ' of Δ belongs to $\operatorname{Con}\mathcal{B}_m$, since $\Delta \wedge \Delta' = \Delta_{B_m}$ ($\mathcal{B}_m$ is the subalgebra corresponding to B_m). Therefore, $\Delta' \le B_m^2$, and since $\Delta \vee \Delta' = A^2$, B_m^2 is also a complement of Δ. An algebra $\mathcal{A}$ is not trivial, hence $|B_m| > 1$. Now, if ρ is a congruence of $\mathcal{A}$ ($\neq A^2$) with a block which is a subalgebra $\mathcal{C}$ of $\mathcal{A}$, then $B_m^2 \vee \Delta \le C^2 \vee \Delta < A^2$, which contradicts the (proved) fact that B_m^2 is a complement of Δ. Finally, $\mathcal{A}$ must have nullary operations, otherwise the empty set would be the smallest subuniverse.

Conversely, suppose that $\mathcal{A}$ has nullary operations and that no congruence of $\mathcal{A}$ has a block which is a proper subalgebra of $\mathcal{A}$. By the first assumption the smallest subuniverse $\mathcal{B}_m$ of $\mathcal{A}$ is nonempty, and if there is no complement of Δ in $\operatorname{Cw}\mathcal{A}$, then $B_m^2 \vee \Delta = \theta < A^2$, and $[B_m]_\theta$ is a proper subalgebra of $\mathcal{A}$. This contradicts the second assumption, and thus Δ must have a complement. $\square$

We proceed with conditions under which the lattice of weak congruences of an algebra is relatively complemented (which means that every interval in this lattice is a complemented lattice itself).

THEOREM 1. *The lattice of weak congruences of an algebra $\mathcal{A}$ is relatively complemented if and only if all of the following conditions are satisfied:*

(1) *$\mathcal{A}$ has at least one nullary operation,*
(2) *no nontrivial congruence of $\mathcal{A}$ has a block which is a subalgebra of $\mathcal{A}$,*
(3) *$\mathcal{A}$ satisfies the CEP and the CIP, and*
(4) *both $\operatorname{Sub}\mathcal{A}$ and $\operatorname{Con}\mathcal{A}$ are relatively complemented lattices.*

PROOF. If $\operatorname{Cw}\mathcal{A}$ is relatively complemented, then the principal ideal ${\downarrow}\Delta$ is standard, and hence Δ is a standard element of this lattice. Being also codistributive, Δ is a neutral element of $\operatorname{Cw}\mathcal{A}$, so $\mathcal{A}$ satisfies both the CEP and the CIP. Since Δ has a complement, by Proposition 1 there are nullary operations in $\mathcal{A}$ and no congruence has a block which is a subalgebra. Obviously, $\operatorname{Sub}\mathcal{A}$ and $\operatorname{Con}\mathcal{A}$ are relatively complemented, as interval sublattices of $\operatorname{Cw}\mathcal{A}$.

Conversely, if $\mathcal{A}$ has nullary operations and no congruence has a block which is a subalgebra, then Δ has a complement in the weak congruence lattice. Further, by the CEP and the CIP, Δ is a neutral element in the lattice $\operatorname{Cw}\mathcal{A}$. Thus, Δ is in the center of this lattice, which is therefore isomorphic with $\operatorname{Sub}\mathcal{A} \times \operatorname{Con}\mathcal{A}$. These two lattices are relatively complemented, hence $\operatorname{Cw}\mathcal{A}$ has the same property. $\square$

Complementedness of weak congruence lattices was characterized in [**9**], as follows.

PROPOSITION 2. [**9**] *An algebra $\mathcal{A}$ has the CEP, the CIP and a complemented weak congruence lattice if and only if it satisfies the following conditions:*

(i) for every subalgebra $\mathcal{B}$, $\operatorname{Con}\mathcal{B}$ is isomorphic with $\operatorname{Con}\mathcal{A}$ under $\rho \mapsto \rho_A$ and

(ii) $\operatorname{Sub}\mathcal{A}$ and $\operatorname{Con}\mathcal{A}$ are complemented lattices. $\square$

The following proposition is closely connected to Theorem 2 below.

PROPOSITION 3. *If the weak congruence lattice of an algebra $\mathcal{A}$ is complemented, then $\operatorname{Sub}\mathcal{A}$ is a complemented lattice. If, in addition, $\mathcal{A}$ satisfies the wCIP, then also $\operatorname{Con}\mathcal{A}$ is complemented.*

PROOF. Suppose that $\operatorname{Cw}\mathcal{A}$ is complemented, then by Proposition 1 the smallest subuniverse B_m of $\mathcal{A}$ is nonempty. Let $\mathcal{C} \in \operatorname{Sub}\mathcal{A}$. The corresponding diagonal relation Δ_C has a complement ρ in $\operatorname{Cw}\mathcal{A}$, hence $\Delta_C \vee \rho = A^2$ and $\Delta_C \wedge \rho = \Delta_{B_m}$. If $\mathcal{D}$ is a subalgebra of $\mathcal{A}$ and $\rho \in \operatorname{Con}\mathcal{D}$, it follows that $\Delta_C \vee \Delta_D = \Delta$ and $\Delta_C \wedge \Delta_D = \Delta_{B_m}$.

Hence, in the lattice $\operatorname{Sub}\mathcal{A}$, we have $\mathcal{C} \vee \mathcal{D} = \mathcal{A}$ and $\mathcal{C} \wedge \mathcal{D} = \mathcal{B}_m$, which proves that $\operatorname{Sub}\mathcal{A}$ is complemented.

Suppose now that $\mathcal{A}$ has the wCIP, and that the complement of $\theta \in \operatorname{Con}\mathcal{A}$ in $\operatorname{Cw}\mathcal{A}$ is θ'. We prove that θ has a complement in $\operatorname{Con}\mathcal{A}$ as well. Note that $\theta \wedge \theta' = \Delta_{B_m}$. Now, the complement of θ in $\operatorname{Con}\mathcal{A}$ is $\Delta \vee \theta'$. Indeed,

$$\theta \vee (\Delta \vee \theta') = (\theta \vee \Delta) \vee \theta' = \theta \vee \theta' = A^2.$$

Further, by the wCIP and by properties of θ'

$$(\Delta \vee \theta') \wedge \theta = \Delta \vee (\theta' \wedge \theta) = \Delta \vee \Delta_{B_m} = \Delta.$$

Hence, $\operatorname{Con}\mathcal{A}$ is a complemented lattice. $\square$

PROPOSITION 4. *If an algebra $\mathcal{A}$ has the CIP, Δ has a complement in the lattice $\operatorname{Cw}\mathcal{A}$, and both $\operatorname{Con}\mathcal{A}$ and $\operatorname{Sub}\mathcal{A}$ are complemented lattices, then also $\operatorname{Cw}\mathcal{A}$ is complemented.*

PROOF. Let ρ be an arbitrary weak congruence of $\mathcal{A}$, such that $\rho \in \operatorname{Con}\mathcal{B}$, for a subalgebra $\mathcal{B}$ of $\mathcal{A}$. If $\theta = \rho \vee \Delta$ in $\operatorname{Cw}\mathcal{A}$, then by the assumption (complementedness of $\operatorname{Con}\mathcal{A}$), there is $\theta' \in \operatorname{Con}\mathcal{A}$, such that $\theta \vee \theta' = A^2$ and $\theta \wedge \theta' = \Delta$.

Let $\mathcal{B}'$ be a complement of $\mathcal{B}$ in $\operatorname{Sub}\mathcal{A}$, which exists by the assumption, and let $\rho' := \theta' \wedge {B'}^2$ (in $\operatorname{Cw}\mathcal{A}$). Note that $\rho' \in \operatorname{Con}\mathcal{B}'$, and that the fact that Δ has a complement (which is, by Proposition 1, B_m^2), implies:

$$\Delta \vee \rho' = \Delta \vee ({B'}^2 \wedge \theta') = (\Delta \vee {B'}^2) \wedge \theta' = A^2 \wedge \theta' = \theta'.$$

We prove that ρ' is a complement of ρ in the lattice $\mathrm{Cw}\,\mathcal{A}$. Since $\mathcal{B} \vee \mathcal{B}' = \mathcal{A}$ in $\mathrm{Sub}\,\mathcal{A}$, we have that $\rho' \vee \rho \in \mathrm{Con}\,\mathcal{A}$. Therefore, $\rho' \vee \rho = (\rho' \vee \Delta) \vee (\rho \vee \Delta) = \theta' \vee \theta = A^2$. Further, by the CIP $\Delta \vee (\rho' \wedge \rho) = (\Delta \vee \rho') \wedge (\Delta \vee \rho) = \theta' \wedge \theta = \Delta$. Hence, $\rho' \wedge \rho$ is a diagonal relation, and since it belongs to $\mathrm{Con}\,\mathcal{B}_m$, it follows that $\rho' \wedge \rho = \Delta_{B_m}$, which completes the proof. □

Summing up, we have proved.

THEOREM 2. *Let $\mathcal{A}$ be an algebra which satisfies the CIP. Then the lattice of weak congruences of $\mathcal{A}$ is complemented if and only if the following conditions are all satisfied:*

(1) *$\mathcal{A}$ has at least one nullary operation;*
(2) *no congruence of $\mathcal{A}$ has a block which is a proper subalgebra of $\mathcal{A}$;*
(3) $\mathrm{Con}\,\mathcal{A}$ *and* $\mathrm{Sub}\,\mathcal{A}$ *are complemented lattices.* □

The following example illustrates the previous proposition.

EXAMPLE 1. We present an algebra which has the CIP (but not the CEP), and whose lattice of weak congruences is complemented. Let $A = \{a, b, c, d\}$ and $\mathcal{A} = (A, *, a, b, c)$, where a, b and c are nullary operations and $*$ is a binary operation defined by the table below. The only subuniverse is $B_m = \{a, b, c\}$. The algebra $\mathcal{A}$ has no nontrivial congruences while B_m has only one: $\rho = \{\{a, b\}, \{c\}\}$. The lattice $\mathrm{Cw}\,\mathcal{A}$ is represented in Figure 1. □

$*$	a	b	c	d
a	b	b	c	c
b	b	a	c	d
c	c	c	a	c
d	c	d	c	d

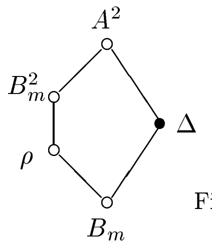

Figure 1

The following consequence of Theorem 1 characterizes algebras having Boolean lattices of weak congruences. This characterization is different from the corresponding result in [**9**].

COROLLARY 1. *The weak congruence lattice of an algebra $\mathcal{A}$ is Boolean if and only if $\mathcal{A}$ has nullary operations, satisfies the CEP and the wCIP, no congruence of $\mathcal{A}$ has a block which is a proper subalgebra of $\mathcal{A}$, and* $\mathrm{Sub}\,\mathcal{A}$ *and* $\mathrm{Con}\,\mathcal{A}$ *are Boolean lattices.*

PROOF. Directly by Theorem 1, since algebras with Boolean lattices of weak congruences satisfy the CEP and the CIP, and hence the wCIP. □

To complete this investigation, we answer the following problem.

PROBLEM 1. *Is relative complementedness of weak congruence lattices hereditary for the main algebraic constructions: subalgebras, homomorphic images and direct products?*

PROPOSITION 5. *If the lattice of weak congruences of $\mathcal{A}$ is relatively complemented, then the same property has the weak congruence lattice of any subalgebra of $\mathcal{A}$.*

PROOF. The lattice $\mathrm{Cw}\,\mathcal{B}$ of a subalgebra $\mathcal{B}$ of $\mathcal{A}$ is an interval sublattice of $\mathrm{Cw}\,\mathcal{A}$, namely, it is the principal ideal $\downarrow B^2$. Hence, it is relatively complemented if $\mathrm{Cw}\,\mathcal{A}$ is. $\square$

As the next assertion shows, homomorphisms preserve relative complements in the weak congruence lattice only under certain conditions.

PROPOSITION 6. *If the weak congruence lattice of an algebra $\mathcal{A}$ is relatively complemented, then the weak congruence lattice of $\mathcal{A}/\theta$, $\theta \in \mathrm{Con}\,\mathcal{A}$, is relatively complemented if and only if* $\mathrm{Sub}\,\mathcal{A}/\theta$ *is a relatively complemented lattice.*

PROOF. Suppose that $\mathrm{Cw}\,\mathcal{A}$ is relatively complemented and $\theta \in \mathrm{Con}\,\mathcal{A}$. Then $\mathrm{Cw}\,\mathcal{A}/\theta$ is isomorphic to the following part of $\mathrm{Cw}\,\mathcal{A}$:

$$\bigcup\{[B^2 \wedge \theta, B^2] \mid \mathcal{B} \in \mathrm{Sub}\,\mathcal{A} \text{ and } B[\theta] = B\},$$

where $B[\theta] := \{x \in A \mid x\,\theta\,b \text{ for some } b \in B\}$. It is straightforward to prove that we indeed obtain a sublattice of $\mathrm{Cw}\,\mathcal{A}$. The smallest subuniverse of $\mathcal{A}/\theta$ is nonempty, since $\mathcal{A}$ also has this property. Its diagonal relation corresponds in $\mathrm{Cw}\,\mathcal{A}$ to $C_m^2 \wedge \theta$, for an appropriate subalgebra $\mathcal{C}_m$ of $\mathcal{A}$. It is straightforward that C_m^2 is a complement of θ in $\mathrm{Cw}\,\mathcal{A}/\theta$. Hence, by Proposition 1, $\mathcal{A}/\theta$ has nullary operations and has no congruence with a class which is its proper subalgebra. The algebra $\mathcal{A}$ has the CEP and the CIP. We prove that $\mathcal{A}/\theta$ has the same properties. Let $\rho, \sigma \in [B^2 \wedge \theta, B^2]$ (this interval in $\mathrm{Cw}\,\mathcal{A}$ represents a congruence lattice of a subalgebra of $\mathcal{A}/\theta$), and suppose $\rho \vee \theta = \sigma \vee \theta$. We also have $\rho \wedge \theta = \sigma \wedge \theta$, and $\rho \wedge \Delta = \sigma \wedge \Delta$. Now, since $\Delta \vee \rho \geq \theta$, we have $\Delta \vee \rho = \theta \vee \rho$, and similarly $\Delta \vee \sigma = \theta \vee \sigma$. Hence, since $\mathcal{A}$ has the CEP, Δ is cancellable and $\rho = \sigma$. Thus $\mathcal{A}/\theta$ has the CEP. To prove the CIP for $\mathcal{A}/\theta$, by the similar arguments as above, since $\mathcal{A}$ has the CIP, we have that for any $\rho \in [B^2 \wedge \theta, B^2]$, $\tau \in [C^2 \wedge \theta, C^2]$, $\theta \vee (\rho \wedge \tau) = \Delta \vee (\rho \wedge \tau) = (\Delta \vee \rho) \wedge (\Delta \vee \tau) = (\theta \vee \rho) \wedge (\theta \vee \tau)$, and the CIP holds. Finally, $\mathrm{Con}\,\mathcal{A}/\theta$ is isomorphic to the principal filter in $\mathrm{Con}\,\mathcal{A}$ and hence in $\mathrm{Con}\,\mathcal{A}/\theta$, which proves that it is relatively complemented. Now it is obvious that relative complementedness of $\mathrm{Sub}\,\mathcal{A}/\theta$ implies the same property of $\mathrm{Cw}\,\mathcal{A}/\theta$.

The converse is obvious. $\square$

In order to examine whether relative complementedness is preserved under taking direct products, consider, for an algebra $\mathcal{A}$, the lattice $\mathrm{Cw}\,\mathcal{A}^2$. Denote by $\mathcal{D}$ a subalgebra of $\mathcal{A}^2$ whose underlying set is the diagonal Δ of A. If $\theta = \{((x,y),(z,t)) \mid x = z\}$, and $\Delta_\Delta = \{((x,x),(x,x)) \mid x \in A\}$, then the interval $[\Delta_\Delta, \theta]$ in $\mathrm{Cw}\,\mathcal{A}^2$ is not complemented. Indeed, Δ_{A^2} belongs to that interval, and does not have a complement in this interval. If Δ' were such a complement, then we would have $\Delta' \in \operatorname{Con}\mathcal{D}$ and $\Delta' \neq \Delta_\Delta$. But $\theta \wedge \Delta^2 = \Delta_\Delta$, hence no element from $\operatorname{Con}\mathcal{D}$, except Δ_Δ, is less than θ, i.e., no element which could be a complement of Δ belongs to the interval $[\Delta_\Delta, \theta]$.

We have thus proved that *relative complementedness of weak congruence lattices is not hereditary for direct products of algebras.*

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