PUBLICATIONS DE L'INSTITUT MATHÉMATIQUE
Nouvelle série, tome 89(103) (2011), 69–75 DOI: 10.2298/PIM1103069U

SEMI-PARALLEL LIGHTLIKE HYPERSURFACES OF INDEFINITE COSYMPLECTIC SPACE FORMS

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Communicated by Darko Milinković

Abstract. We study the semiparallel lightlike hypersurface of an indefinite cosymplectic space forms which are tangent to the structure vector field.

1. Introduction

In the theory of submanifolds of semi-Riemannian manifolds it is interesting to study the geometry of lightlike submanifolds due to the fact that the intersection of normal vector bundle and the tangent bundle is nontrivial making it more interesting and remarkably different from the study of nondegenerate submanifolds. The geometry of lightlike hypersurfaces and submanifolds of indefinite Kaehler manifolds was studied by Duggal and Bejancu [5]. On the other hand, lightlike hypersurfaces of indefinite Sasakian manifolds was studied in [3, 6], whereas lightlike hypersurfaces in indefinite cosymplectic space form was studied in [7].

The basic Gauss, Codazzi–Mainardi and Ricci equations give that the extrinsic conditions parallel, semiparallel and pseudo-parallel imply the correspondent intrinsic conditions symmetry, semisymmetry and pseudo-symmetry, respectively [1].

We study semiparallel lightlike hypersurface of an indefinite cosymplectic space form and we prove:

Theorem 1.1. *Let M be a semiparallel lightlike hypersurface of an indefinite cosymplectic space form $\bar{M}(c)$ of constant curvature c, with $\xi \in TM$. Then either $c = 0$, or M is $(\bar{\phi}(TM^{\perp}), D \oplus D')$ mixed totally geodesic. Moreover, if $c = 0$, then either M is totally geodesic or $C(E, A^{*}_{E}PX) = 0$, for any $X \in \Gamma(TM)$.*

2010 *Mathematics Subject Classification:* Primary 53C15, 53C40, 53C50, 53D15.

Key words and phrases: degenerate metric, cosymplectic manifold.

Acknowledgement: This research is partly supported by the University Grants Commission (UGC), India, under a Major Research Project No. SR. 36-321/2008. The second author would like to thank the UGC for providing the financial support to pursue this research work.

2. Preliminaries

An odd-dimensional semi-Riemannian manifold $\bar{M}$ is said to be an indefinite almost contact metric manifold if there exist structure tensors $\{\bar{\phi}, \xi, \eta, \bar{g}\}$, where $\bar{\phi}$ is a (1,1) tensor field, ξ a vector field, η a 1-form and $\bar{g}$ is the semi-Riemannian metric on $\bar{M}$ satisfying

$$\bar{\phi}^2\bar{X} = -\bar{X} + \eta(\bar{X})\xi, \quad \eta \circ \bar{\phi} = 0, \quad \bar{\phi}\xi = 0, \quad \eta(\xi) = 1 \tag{2.1}$$
$$\bar{g}(\bar{\phi}\bar{X}, \bar{\phi}\bar{Y}) = \bar{g}(\bar{X}, \bar{Y}) - \varepsilon\eta(\bar{X})\eta(\bar{Y}), \quad \eta(\bar{X}) = \varepsilon\bar{g}(\bar{X}, \xi), \quad \bar{g}(\xi, \xi) = \varepsilon, \quad \varepsilon = \pm 1$$

for any $\bar{X}, \bar{Y} \in \Gamma(T\bar{M})$, where $\Gamma(T\bar{M})$ denotes the Lie algebra of vector fields on $\bar{M}$.

An indefinite almost contact metric manifold $\bar{M}$ is called an indefinite cosymplectic manifold if [4] $(\bar{\nabla}_{\bar{X}}\bar{\phi})\bar{Y} = 0$, and $\bar{\nabla}_{\bar{X}}\xi = 0$ for any $\bar{X}, \bar{Y} \in T\bar{M}$, where $\bar{\nabla}$ denotes the Levi–Civita connection on $\bar{M}$.

A plane section Π in $T_x\bar{M}$ of a cosymplectic manifold $\bar{M}$ is called a $\bar{\phi}$-section if it is spanned by a unit vector $\bar{X}$ orthogonal to ξ and $\bar{\phi}\bar{X}$, where $\bar{X}$ is a non-null vector field on $\bar{M}$. The sectional curvature $K(\Pi)$ with respect to Π determined by $\bar{X}$ is called a $\bar{\phi}$-sectional curvature. If $\bar{M}$ has a $\bar{\phi}$-sectional curvature c which does not depend on the $\bar{\phi}$-section at each point, then c is a constant in $\bar{M}$ and $\bar{M}$ is called an indefinite cosymplectic space form, which is denoted by $\bar{M}(c)$. The curvature tensor $\bar{R}$ of $\bar{M}(c)$ is given by [4]

$$\begin{aligned}\bar{R}(\bar{X}, \bar{Y})\bar{Z} = \frac{c}{4}\big\{&\bar{g}(\bar{Y}, \bar{Z})\bar{X} - \bar{g}(\bar{X}, \bar{Z})\bar{Y} + \eta(\bar{X})\eta(\bar{Z})\bar{Y} - \eta(\bar{Y})\eta(\bar{Z})\bar{X}\\ &+\bar{g}(\bar{X}, \bar{Z})\eta(\bar{Y})\xi - \bar{g}(\bar{Y}, \bar{Z})\eta(\bar{X})\xi + \bar{g}(\bar{\phi}\bar{Y}, \bar{Z})\bar{\phi}\bar{X}\\ &-\bar{g}(\bar{\phi}\bar{X}, \bar{Z})\bar{\phi}\bar{Y} - 2\bar{g}(\bar{\phi}\bar{X}, \bar{Y})\bar{\phi}\bar{Z}\,\big\}\end{aligned} \tag{2.2}$$

for any $\bar{X}, \bar{Y}, \bar{Z} \in \Gamma(T\bar{M})$.

Let (M, g) be a hypersurface of a $(2n+1)$-dimensional semi-Riemannian manifold $(\bar{M}, \bar{g})$ with index s, $0 < s < 2n+1$ and $g = \bar{g}_{|M}$. Then M is a lightlike hypersurface of $\bar{M}$ if g is of constant rank $(2n-1)$ and the normal bundle $TM^{\perp}$ is a distribution of rank 1 on M [5]. A nondegenerate complementary distribution $S(TM)$ of rank $(2n-1)$ to $TM^{\perp}$ in TM, that is, $TM = TM^{\perp} \perp S(TM)$, is called screen distribution. The following result (cf. [5, Theorem 1.1, p. 79]) has an important role in studying the geometry of lightlike hypersurfaces.

Theorem 2.1. *Let $(M, g, S(TM))$ be a lightlike hypersurface of $\bar{M}$. Then, there exists a unique vector bundle* $\mathrm{tr}(TM)$ *of rank* 1 *over M such that for any nonzero section E of $TM^{\perp}$ on a coordinate neighbourhood $U \subset M$, there exists a unique section N of* $\mathrm{tr}(TM)$ *on U satisfying* $\bar{g}(N, E) = 1$ *and* $\bar{g}(N, N) = \bar{g}(N, W) = 0$*, for each $W \in \Gamma(S(TM)|_U)$.*

Then, we have the following decomposition:

$$TM = S(TM) \perp TM^{\perp}, \qquad T\bar{M} = S(TM) \perp (TM^{\perp} \oplus \mathrm{tr}(TM)). \tag{2.3}$$

Throughout this paper, all manifolds are supposed to be paracompact and smooth. We denote by $\Gamma(E)$ the smooth sections of the vector bundle E, and by

$\perp$ and $\oplus$ the orthogonal and the nonorthogonal direct sum of two vector bundles, respectively.

Let $\bar{\nabla}$, ∇ and ∇^t denote the linear connections on $\bar{M}$, M and vector bundle $\mathrm{tr}(TM)$, respectively. Then, the Gauss and Weingarten formulae are given by

$$\bar{\nabla}_X Y = \nabla_X Y + h(X,Y), \quad \text{for all } X, Y \in \Gamma(TM), \tag{2.4}$$

$$\bar{\nabla}_X V = -A_V X + \nabla^t_X V, \quad \text{for all } V \in \Gamma(\mathrm{tr}(TM)), \tag{2.5}$$

where $\{\nabla_X Y, A_V X\}$ and $\{h(X,Y), \nabla^t_X V\}$ belong to $\Gamma(TM)$ and $\Gamma(\mathrm{tr}(TM))$, respectively and A_V is the shape operator of M with respect to V. Moreover, in view of decomposition (2.3), equations (2.4) and (2.5) take the form

$$\bar{\nabla}_X Y = \nabla_X Y + B(X,Y)N \tag{2.6}$$

$$\bar{\nabla}_X N = -A_N X + \tau(X)N \tag{2.7}$$

for any $X, Y \in \Gamma(TM)$ and $N \in \Gamma(\mathrm{tr}(TM))$, where $B(X,Y)$ and $\tau(X)$ are local second fundamental form and a 1-form on U, respectively. It follows that

$$B(X,Y) = \bar{g}(\bar{\nabla}_X Y, E) = \bar{g}(h(X,Y), E), B(X,E) = 0, \quad \tau(X) = \bar{g}(\nabla^t_X N, E).$$

Let P denote the projection of TM on $S(TM)$ and ∇^*, ∇^{*t} denote the induced linear connections on $S(TM)$ and $TM^\perp$, respectively. Then from the decomposition of tangent bundle of lightlike hypersurface, we have

$$\nabla_X PY = \nabla^*_X PY + h^*(X, PY) \tag{2.8}$$

$$\nabla_X E = -A^*_E X + \nabla^{*t}_X E \tag{2.9}$$

for any $X, Y \in \Gamma(TM)$ and $E \in \Gamma(TM^\perp)$, where h^*, A^* are the second fundamental form and the shape operator of distribution $S(TM)$ respectively.

By direct calculations using Gauss–Weingarten formulae (2.8) and (2.9), we find

$$g(A_N Y, PW) = \bar{g}(N, h^*(Y, PW)), \qquad \bar{g}(A_N Y, N) = 0; \tag{2.10}$$

$$g(A^*_E X, PY) = \bar{g}(E, h(X, PY)), \qquad \bar{g}(A^*_E X, N) = 0; \tag{2.11}$$

for any $X, Y, W \in \Gamma(TM)$, $E \in \Gamma(TM^\perp)$ and $N \in \Gamma(\mathrm{tr}(TM))$.

Locally, we define on U

$$C(X, PY) = \bar{g}(h^*(X, PY), N), \quad \text{and} \quad \lambda(X) = \bar{g}(\nabla^{*t}_X E, N). \tag{2.12}$$

Hence, $h^*(X, PY) = C(X, PY)E$, and $\nabla^{*t}_X E = \lambda(X)E$. On the other hand, by using (2.6), (2.7), (2.9) and (2.12), we obtain

$$\lambda(X) = \bar{g}(\nabla_X E, N) = \bar{g}(\bar{\nabla}_X E, N) = -\bar{g}(E, \bar{\nabla}_X N) = -\tau(X).$$

Thus, locally (2.8) and (2.9) become

$$\nabla_X PY = \nabla^*_X PY + C(X, PY)E, \quad \text{and} \quad \nabla_X E = -A^*_E X - \tau(X)E.$$

Finally, (2.10) and (2.11), locally become

$$g(A_N Y, PW) = C(Y, PW), \quad \bar{g}(A_N Y, N) = 0;$$
$$g(A^*_E X, PY) = B(X, PY), \quad \bar{g}(A^*_E X, N) = 0.$$

In general, the induced connection ∇ on M is not a metric connection. Since $\bar{\nabla}$ is a metric connection, we have

$$0 = (\bar{\nabla}_X \bar{g})(Y, Z) = X(\bar{g}(Y, Z)) - \bar{g}(\bar{\nabla}_X Y, Z) - \bar{g}(Y, \bar{\nabla}_X Z).$$

If $\bar{R}$ and R are the curvature tensors of $\bar{\nabla}$ and ∇, then using (2.6) in the equation $\bar{R}(X,Y)Z = \bar{\nabla}_X \bar{\nabla}_Y Z - \bar{\nabla}_Y \bar{\nabla}_X Z - \bar{\nabla}_{[X,Y]} Z$, we obtain

$$\begin{aligned}\bar{R}(X,Y)Z &= R(X,Y)Z + B(X,Z)A_N Y - B(Y,Z)A_N X \\ &\quad + \big\{(\nabla_X B)(Y,Z) - (\nabla_Y B)(X,Z) + \tau(X)B(Y,Z) - \tau(Y)B(X,Z)\big\}N.\end{aligned} \tag{2.13}$$

A hypersurface M is semiparallel if its second fundamental form h satisfies,

$$(R(X,Y) \cdot h)(X_1, X_2) = -h(R(X,Y)X_1, X_2) - h(X_1, R(X,Y)X_2) = 0 \tag{2.14}$$

for any $X, Y, X_1, X_2 \in \Gamma(TM)$, where R is the curvature tensor field of M.

3. Proof of the theorem

Let $(\bar{M}, \bar{\phi}, \xi, \eta, \bar{g})$ be an indefinite cosymplectic manifold and (M, g) be its lightlike hypersurface, tangent to the structure vector field ξ with $\bar{g}(\xi, \xi) = \varepsilon = +1$. If E is a local section of $TM^{\perp}$, then $\bar{g}(\bar{\phi}E, E) = 0$ implies that $\bar{\phi}E$ is tangent to M. Thus $\bar{\phi}(TM^{\perp})$ is a distribution on M of rank 1 such that $\bar{\phi}(TM^{\perp}) \cap TM^{\perp} = \{0\}$. This enables us to choose a screen distribution $S(TM)$ such that it contains $\bar{\phi}(TM^{\perp})$ as vector subbundle.

Now, we consider a local section N of $\mathrm{tr}(TM)$. Then $\bar{\phi}N$ is tangent to M and belongs to $S(TM)$ as $\bar{g}(\bar{\phi}N, E) = -\bar{g}(N, \bar{\phi}E) = 0$ and $\bar{g}(\bar{\phi}N, N) = 0$. From (2.1), we have

$$\bar{g}(\bar{\phi}N, \bar{\phi}E) = \bar{g}(N, E) - \eta(N)\eta(E) = \bar{g}(N, E) = 1.$$

Therefore, $\bar{\phi}(TM^{\perp}) \oplus \bar{\phi}(\mathrm{tr}(TM))$ is a direct sum but not orthogonal and is a non-degenerate vector subbundle of $S(TM)$ of rank 2.

It is known [2] that if M is tangent to structure vector field ξ, then ξ belongs to $S(TM)$. Since $\bar{g}(\bar{\phi}E, \xi) = \bar{g}(\bar{\phi}N, \xi) = 0$, there exists a non degenerate invariant distribution D_0 of rank $(2n-4)$ on M such that

$$S(TM) = \big\{\bar{\phi}(TM^{\perp}) \oplus \bar{\phi}(\mathrm{tr}(TM))\big\} \perp D_0 \perp \langle\xi\rangle \quad \text{and} \quad \bar{\phi}(D_0) = D_0, \tag{3.1}$$

where $\langle\xi\rangle = \operatorname{span}\xi$. Moreover, from (2.3) and (3.1), we obtain

$$TM = \big\{\bar{\phi}(TM^{\perp}) \oplus \bar{\phi}(\mathrm{tr}(TM))\big\} \perp D_0 \perp \langle\xi\rangle \perp TM^{\perp}.$$

Now, we consider the distributions D and D' on M as follows

$$D = TM^{\perp} \perp \bar{\phi}(TM^{\perp}) \perp D_0, \quad D' = \bar{\phi}(\mathrm{tr}(TM)).$$

Then D is invariant under $\bar{\phi}$ and $TM = D \oplus D' \perp \langle\xi\rangle$.

If P_1 and Q denote the projection morphisms of TM on D and D' and $U = -\bar{\phi}N$, $V = -\bar{\phi}E$ are local lightlike vectors, respectively, then we write

$$X = P_1 X + QX + \eta(X)\xi \tag{3.2}$$

for $X \in \Gamma(TM)$, where $QX = u(X)U$, and u is a differential 1-form locally defined on M by $u(\cdot) = g(V, \cdot)$. From (3.1) and (3.2), we obtain $\bar{\phi}X = \phi X + u(X)N$ and

$\phi^2 X = -X + \eta(X)\xi + u(X)U$, for each $X \in \Gamma(TM)$, where ϕ is a tensor field of type $(1,1)$ defined on M by $\phi X = \bar{\phi} P_1 X$.

Putting (2.2), (2.6), (2.13) into (2.14), by a straightforward calculation we obtain

$$\begin{aligned}
0 = \frac{c}{4}\big[\, & g(Y,X_1)B(X,X_2) - g(X,X_1)B(Y,X_2) + \eta(X)\eta(X_1)B(Y,X_2) \\
& - \eta(Y)\eta(X_1)B(X,X_2) + \bar{g}(\bar{\phi}Y,X_1)B(\phi X,X_2) \\
& - \bar{g}(\bar{\phi}X,X_1)B(\phi Y,X_2) - 2\bar{g}(\bar{\phi}X,Y)B(\phi X_1,X_2)\big] \\
& - B(X,X_1)B(A_N Y,X_2) + B(Y,X_1)B(A_N X,X_2) \\
& + \frac{c}{4}\big[\, g(Y,X_2)B(X,X_1) - g(X,X_2)B(Y,X_1) + \eta(X)\eta(X_2)B(Y,X_1) \\
& - \eta(Y)\eta(X_2)B(X,X_1) + \bar{g}(\bar{\phi}Y,X_2)B(\phi X,X_1) \\
& - \bar{g}(\bar{\phi}X,X_2)B(\phi Y,X_1) - 2\bar{g}(\bar{\phi}X,Y)B(\phi X_2,X_1)\big] \\
& - B(X,X_2)B(A_N Y,X_1) + B(Y,X_2)B(A_N X,X_1).
\end{aligned}$$

Putting above $X = E$ and using the fact that $B(E,\cdot) = 0$, we get

$$\begin{aligned}
0 = {} & \frac{c}{4}\big[\bar{g}(\bar{\phi}Y,X_1)B(\phi E,X_2) + u(X_1)B(\phi Y,X_2) + 2u(Y)B(\phi X_1,X_2)\big] \\
& + B(Y,X_1)B(A_N E,X_2) + \frac{c}{4}\big[\bar{g}(\bar{\phi}Y,X_2)B(\phi E,X_1) + u(X_2)B(\phi Y,X_1) \\
& + 2u(Y)B(\phi X_2,X_1)\big] + B(Y,X_2)B(A_N E,X_1).
\end{aligned} \tag{3.3}$$

Putting $X_2 = E$ into (3.3) we get $\frac{3}{4}cu(Y)B(V,X_1) = 0$. If we put here $Y = U$, we find

$$\frac{3}{4}cB(V,X_1) = 0. \tag{3.4}$$

From (3.4), we get $c = 0$ as $B(V,X_1) \neq 0$, for each $X_1 \in \Gamma(D \oplus D')$. If $c \neq 0$, then (3.4) implies that $B(V,X_1) = 0$, for each $X_1 \in \Gamma(D \oplus D')$. Hence M is $(\bar{\phi}(TM^\perp), D \oplus D')$-mixed totally geodesic.

On the other hand, suppose that $c = 0$; then from (3.3), by putting $X_1 = X_2$, we obtain

$$B(Y,X_1)B(A_N E,X_1) = 0. \tag{3.5}$$

If $B(Y,X_1) = 0$ for each $Y, X_1 \in \Gamma(TM)$, then M is totally geodesic. If $B(Y,X_1) \neq 0$, then (3.5) imply that $B(A_N E,X_1) = 0$, that is $C(E, A^*_E P X_1) = 0$, for any $X_1 \in \Gamma(TM)$. This finishes the proof of our Theorem. □

From Theorem 1.1 and the fact that $C(E, A^*_E PX) = \mathrm{Ric}(E,X)$, $X \in \Gamma(TM)$, where Ric denotes the Ricci tensor of M, we have the following characterization (cf. [5, Theorem 2.2, p. 88]):

COROLLARY. *Let $(M, g, S(TM))$ be a semiparallel lightlike hypersurface of an indefinite cosymplectic space form $\bar{M}(c)$ of constant curvature $c = 0$, with $\xi \in TM$, such that* $\mathrm{Ric}(E,X) \neq 0$*, for any $X \in \Gamma(TM)$ and $E \in \Gamma(TM^\perp)$. Then following assertions are equivalent:*

(i) *$A^*_W X = 0$, for any $W \in \Gamma(TM^\perp)$ and $X \in \Gamma(TM)$.*

(ii) *There exists a unique torsion free metric connection ∇ induced by $\bar{\nabla}$ on M.*

(iii) $TM^{\perp}$ *is a parallel distribution with respect to* ∇.
(iv) $TM^{\perp}$ *is a killing distribution on* M.

Hereafter, $(R_q^{2m+1}, \bar{\phi}, \xi, \eta, \bar{g})$ will denote the manifold R_q^{2m+1} with its usual cosymplectic structure given by

$$\eta = dz, \qquad \xi = \partial z,$$
$$\bar{g} = \eta \otimes \eta - \sum_{i=1}^{q/2} dx^i \otimes dx^i + dy^i \otimes dy^i + \sum_{i=q}^{m} dx^i \otimes dx^i + dy^i \otimes dy^i,$$
$$\bar{\phi}\left(\sum_{i=1}^{m}(X_i \partial x^i + Y_i \partial y^i) + Z \partial z\right) = \sum_{i=1}^{m}(Y_i \partial x^i - X_i \partial y^i),$$

where (x^i, y^i, z) are Cartesian coordinates.

Example. Let $\bar{M} = (R_2^7, \bar{g})$ be a semi-Euclidean space, where $\bar{g}$ is of signature $(-,+,+,-,+,+,+)$ with respect to the canonical basis

$$\{\partial x_1, \partial x_2, \partial x_3, \partial y_1, \partial y_2, \partial y_3, \partial z\}.$$

Consider the hypersurface M of R_2^7, defined by

$$X(u, v, \theta_1, \theta_2, s, t) = (u, u, v, \theta_1, \theta_2, s, t).$$

Then a local frame of TM is given by

$$\begin{aligned} &Z_1 = \partial x_1 + \partial x_2, \ Z_2 = \partial x_3, \ Z_3 = \partial y_1, \\ &Z_4 = \partial y_2, \qquad\quad\;\; Z_5 = \partial y_3, \ Z_6 = \xi = \partial z. \end{aligned}$$

Hence, $TM^{\perp} = \operatorname{span}\{Z_1\}$ and $\bar{\phi}(TM^{\perp}) = \operatorname{span}\{-Z_3 - Z_4\}$ which implies that $\bar{\phi}(TM^{\perp}) \in \Gamma(S(TM))$. Thus $D = TM^{\perp} \perp \bar{\phi}(TM^{\perp}) \perp D_0$ is invariant under $\bar{\phi}$, where $D_0 = \operatorname{span}\{Z_2, Z_5\}$. Now, $\operatorname{tr}(TM)$ is spanned by $N = \frac{1}{2}(-\partial x_1 + \partial x_2)$ and $D' = \bar{\phi}(\operatorname{tr}(TM)) = \operatorname{span}\{\frac{1}{2}(Z_3 - Z_4)\}$. Hence $TM = D \oplus D' \perp \langle \xi \rangle$.

Using (2.4) and (2.5) we obtain

$$h(Z_i, Z_j) = 0, \quad \text{and} \quad \bar{\nabla}_{Z_i} N = 0, \quad \text{for } i, j = 1, \ldots, 6. \tag{3.6}$$

From (3.6) and (2.14), it is easy to see that M is semiparallel hypersurface of $\bar{M}$. Moreover, using (3.6), M is totally geodesic hypersurface and $c = 0$ as $\bar{M} = R_2^7$ is a semi-Euclidean space, which supports Theorem 1.1.

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(Received 21 11 2009)
(Revised 02 12 2010)