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DOUBLY STOCHASTIC COMPOUND POISSON PROCESSES IN EXTREME VALUE THEORY

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Abstract: For some linear models, chain-dependent sequences and doubly stochastic max-autoregressive processes, which do not satisfy the long range dependence condition $\Delta(u_n)$ from Hsing *et al.* ([7]), the sequence $\{S_n\}_{n\geq 1}$, of point processes of exceedances of a real level u_n by $X_1, ..., X_n$, $n \geq 1$, converges in distribution to a compound Poisson process with stochastic intensity.

These examples illustrate the main result of this paper: for sequences $\{X_n\}_{n\geq 1}$ that conditional on a random variable X satisfy the usual dependence conditions in the extreme value theory, we obtain the convergence of $\{S_n\}$ to a point process whose distribution is a mixture of distributions of compound Poisson processes. Such result permits the identification of a class of sequences for which the extremal behaviour can be described by mixtures of extreme value distributions.

1 – Introduction

Let $\{u_n\}_{n\geq 1}$ be a sequence of real numbers and $\{X_n\}_{n\geq 1}$ a sequence of random variables such that, for some random variable X, every family of conditional distributions given X,

$$\mathcal{P}_x = \Big\{P_{(X_{i_1},...X_{i_n})}(\cdot\,|\,X{=}x)\colon\ n \geq 1,\ i_1{<}\cdots{<}i_n\Big\} \quad (x \text{ in the support of } X)$$

satisfies the dependence condition $\Delta(u_n)$ introduced in Hsing *et al.* ([7]). That

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is, if $\{X_n^x\}_{n\geq 1}$ is a sequence with distribution determined by $\mathcal{P}_x$ then

$$\alpha_{n,\ell} = \alpha_{n,\ell}(x) = \sup\Big\{\Big|P(A,B) - P(A)\,P(B)\Big| : \\ A \in \mathcal{B}_1^k(u_n),\ B \in \mathcal{B}_{k+\ell}^n(u_n),\ 1 \leq k \leq n-\ell\Big\},\quad 1 \leq \ell \leq n-1,$$

where $\mathcal{B}_i^j(u_n)$ denotes the σ-field generated by $\{X_s^x \leq u_n\}$, $i \leq s \leq j$,

satisfies

$$\alpha_{n,\ell_n} \underset{n\to\infty}{\longrightarrow} 0, \qquad \text{for some sequence } \ \ell_n = o(n) \ .$$

From the results in Hsing *et al.* ([7]) it follows that, if $\{X_n^x\}_{n\geq 1}$ is a stationary sequence and the sequence of point processes of exceedances

$$S_n[X_n^x, u_n](\cdot) = \sum_{i=1}^{n} 1_{\{X_i^x > u_n\}}\, \delta_{i/n}(\cdot)\,, \qquad n \geq 1 \ ,$$

converges in distribution to a point process S^x, then S^x is a compound Poisson process $S[\nu,\Pi]$ with Laplace transform

$$L_{S^x}(f) = \exp\Big(-\nu \int_{[0,1]} \Big(1 - \sum_{k=1}^{\infty} e^{-k f(y)}\, \Pi(k)\Big)\, dy\Big) \ ,$$

where $\nu = \nu(x)$ is a positive constant and $\Pi = \Pi^{(x)}$ is a distribution for the multiplicities.

The most general results (Nandagopalan ([14]), Nandagopalan *et al.* ([15])) guarantee that for a non stationary sequence $\{X_n^x\}_{n\geq 1}$ we can also find in the limit a compound process where Poisson events have a finite intensity measure μ and the distributions of multiplicities $\{\Pi_y\}_{y\in[0,1]}$ depend on the position y of the atoms, provided that some additional assumptions of equicontinuity and uniform asymptotic negligibility are satisfied.

Stronger results can be obtained if $\{u_n\}$ is a sequence of normalized levels, $\{u_n = u_n(\tau)\}_{n\geq 1}$, with $\tau = \tau(x)$, for $\{X_n^x\}_{n\geq 1}$, that is, if for each $a \in [0,1]$, it holds

$$\sum_{i=1}^{[na]} P(X_i^x > u_n) \underset{n\to\infty}{\longrightarrow} a\,\tau \ .$$

If $\{X_n^x\}_{n\geq 1}$ is a stationary sequence and if, for some $\tau_0 > 0$, $\{S_n[X_n^x, u_n(\tau_0)]\}_{n\geq 1}$ converges in distribution, then for each $\tau > 0$, $\{S_n[X_n^x, u_n(\tau)]\}_{n\geq 1}$ converges to $S[\theta\,\tau, \Pi]$, with $\theta = \frac{\nu_0}{\tau_0}$ and Π independent of τ.

This result of Hsing *et al.* ([7]) can be applied to some simple forms of non-stationarity like periodic sequences (Alpuim ([1]), Ferreira ([5])) and quasi-stationary sequences (Turkman ([16])).

A significant theory of point processes of rare events, under long range and local dependence conditions, is available in the recent literature and can be applied to obtain the asymptotic distribution S^x of $S_n[X_n^x, u_n]$ (Hüsler, J. ([9]), Falk *et al.* ([3]), Hüsler, J. and Schmidt, M. ([10])). A simple application of the dominated convergence theorem enables us to conclude that $S_n[X_n, u_n]$ will converge to a point process whose distribution is a mixture of the distributions of the processes S^x. Such point process exists when, for each simple function f, the Laplace transform of S^x on f, $\phi(x) = L_{S^x}(f)$ is a measurable function (Kallenberg, [11]).

We begin in section 2 with a general result on the convergence of $\{S_n\}$ to a mixture. As a corollary, when $\{X_n\}$ conditional on realizations of X, is stationary, we obtain a sufficient condition for the convergence of $S_n[X_n, u_n]$ to a doubly stochastic compound Poisson process, *i.e.*, a point process whose distribution is a mixture of the distributions of $S[\nu(x), \Pi]$, the intensity measure being regulated by X.

As applications we shall study the point process of exceedances of u_n generated by the linear sequences $X_n = Y_n + X$ by some sequences whose finite distributions are determined from the values of a sequence of discrete random variables $X = \{J_n\}_{n\geq 1}$ and, finally, exceedances by the max-autoregressive sequences $X_n = \max(Y_n, Y_{n-1}, ..., Y_{n-X})$.

The sequences to which we apply the results of this paper do not satisfy, in general, the condition $\Delta(u_n)$. However, on what concerns the local restrictions on rapid oscillations of $\{X_n\}$ (conditions $D'(u_n)$ from Leadbetter ([12]), $D''(u_n)$ from Leadbetter and Nandagopalan ([13]), $D^{(k)}(u_n)$ from Chernick *et al.* ([2]), $\widetilde{D}^{(k)}(u_n)$ from Ferreira ([5])), they can be satisfied provided the analogous condition holds for each sequence $\{X_n^x\}_{n\geq 1}$.

We say that the condition $\widetilde{D}^{(k)}(u_n)$ holds, for a stationary sequence $\{X_n^x\}$ satisfying $\Delta(u_n)$, if k is the minimum positive integer for which there exists a sequence of positive integers $\{k_n\}_{n\geq 1}$ satisfying

$$k_n \underset{n\to\infty}{\longrightarrow} \infty, \quad k_n \ell_n / n \underset{n\to\infty}{\longrightarrow} 0, \quad k_n\, \alpha_{n,\ell_n} \underset{n\to\infty}{\longrightarrow} 0, \quad k_n\, P(X_1^x > u_n) \underset{n\to\infty}{\longrightarrow} 0$$

and

$$(1.1) \quad s_n^{(k)} = n \sum_{2\leq j_1 < \cdots < j_k \leq [n/k_n]-1} P\Big(X_1^x > u_n,\ \bigcap_{i=1}^{k} \Big\{X_{j_i}^x \leq u_n < X_{j_i+1}^x\Big\}\Big) \underset{n\to\infty}{\longrightarrow} 0\ .$$

When $k = 1$ we get the condition $D''(u_n)$ from Leadbetter and Nandagopalan ([13]).

The condition $D^{(k)}(u_n)$, from Chernick *et al.* ([2]), holds for a stationary sequence $\{X_n^x\}$ if

$$\lim_{n\to\infty} n \sum_{j=k+1}^{[n/k_n]} P\Big(X_1^x > u_n,\ X_2^x \le u_n,\ ...,\ X_k^x \le u_n,\ X_j^x > u_n\Big) = 0 \ . \tag{1.2}$$

These local dependence conditions enable us to obtain $\nu(x)$ and $\Pi^{(x)}$, in the limiting compound Poisson process S^x, from certain limiting probabilities easy to compute, and also give criteria for the existence of the parameter $\theta = \theta(x)$, the extremal index of $\{X_n^x\}$.

By applying Fatou's lemma, if (1.1) or (1.2) holds for each of the sequences $\{X_n^x\}$, then $\{X_n\}$ can satisfy the analogous convergence, which gives information about the episodes of high values of $\{X_n\}$.

2 – Main result and examples

Proposition 2.1. *Let $\{X_n\}_{n\ge1}$ be a sequence of random variables for which there exists a random variable X such that, for each x in the support of X, the sequence $\{X_n^x\}_{n\ge1}$ with distribution determined by $\mathcal{P}_x$ satisfies $S_n[X_n^x, u_n] \xrightarrow[n\to\infty]{d} S^x$. Suppose that, for each continuous, positive and with compact support function f, $\phi(x) = L_{S^x}(f)$ is a measurable function.*

Then $\{S_n[X_n, u_n]\}_{n\ge1}$ converges, in distribution, to a point process whose distribution is a mixture of the distributions of S^x regulated by the distribution of X.

Proof. It is sufficient to prove that, for any finite number of disjoint intervals $I_1, ..., I_k$ in $[0,1]$ and non negative integers $s_1, ..., s_k$, it holds

$$P\Big(\bigcap_{j=1}^{k} \Big\{S_n[X_n, u_n](I_j) = s_j\Big\}\Big) \underset{n\to\infty}{\longrightarrow} \int P\Big(\bigcap_{j=1}^{k} \Big\{S^x(I_j) = s_j\Big\}\Big)\, dP_X(x) \ . \tag{2.1}$$

Since

$$\begin{aligned} P\Big(\bigcap_{j=1}^{k} \Big\{S_n[X_n, u_n](I_j) = s_j\Big\}\Big) &= \int P\Big(\bigcap_{j=1}^{k} \Big\{S_n[X_n, u_n](I_j) = s_j\Big\} \,\Big|\, X{=}x\Big)\, dP_X(x) \\ &= \int P\Big(\bigcap_{j=1}^{k} \Big\{S_n[X_n^x, u_n](I_j) = s_j\Big\}\Big)\, dP_X(x) \ , \end{aligned}$$

then, by using the dominated convergence theorem, we conclude (2.1). ∎

If $\{X_n^x\}$ is a stationary sequence and satisfies the condition $\Delta(u_n)$ then S^x is a compound Poisson process $S[\nu(x),\Pi^{(x)}]$. Furthermore, if we suppose that $u_n = u_n(\tau(x))$ for $\{X_n^x\}$, *i.e.* $\{u_n\}_{n\geq 1}$ is a sequence of normalized levels for $\{X_n^x\}_{n\geq 1}$, then $\nu(x) = \theta(x)\,\tau(x)$, with $0 \leq \theta(x) \leq 1$.

In some examples we find $\Pi^{(x)}$ and $\theta(x)$ independent of x. For instance, if for each x, $\{X_n^x\}$ satisfies the condition $D'(u_n)$, in Leadbetter ([12]), that is

$$\limsup_{n\to\infty} n \sum_{j=2}^{[n/k]} P\Big(X_1^x > u_n,\ X_j^x > u_n\Big) \underset{k\to\infty}{\longrightarrow} 0\ ,$$

then $\theta(x) = 1$ and $\Pi(x)(1) = 1$. By applying the Proposition 2.1, we conclude that in these cases $S_n[X_n, u_n]$ converges in distribution to a Cox process with stochastic intensity $\tau(X)$.

The Proposition 2.1 is too general to give an insight into the limiting point process. The following application is a more useful result since in special cases of interest the limiting point process S^x is a compound Poisson process.

Corollary 2.1. *Let $\{X_n\}$ be a sequence of random variables for which there exists a random variable X such that, for each x in the support of X, the sequence $\{X_n^x\}_{n\geq 1}$ with distribution determined by $\mathcal{P}_x$ satisfies $S_n[X_n^x, u_n] \xrightarrow[n\to\infty]{d} S[\theta\,\tau(x),\Pi]$, where $\tau(x)$ is a measurable function.*

Then $\{S_n[X_n, u_n]\}_{n\geq 1}$ converges, in distribution, to a doubly stochastic compound Poisson process with intensity $\theta\,\tau(X)$ and multiplicity distribution Π. ∎

In the following we present two examples illustrating the above corollary. We shall maintain the notation X for the random variable chosen for conditioning.

We first give a limit distribution for the point process of exceedances generated by a linear model $X_n = Y_n + X$, $n \geq 1$, where $\{Y_n\}$ is a periodic sequence independent of the variable X.

The second example concerns a sequence $\{X_n\}$ whose finite distributions are determined by a sequence of discrete random variables $X = \{J_n\}_{n\geq 1}$.

The asymptotic distribution of S_n in these examples has already been obtained by specific methods (Ferreira ([4]), Turkman and Duarte ([17])). Our aim is to present one single methodology supported by the above results.

Example 2.1. Let T be a positive integer and Y_n, $n \geq 1$, independent exponential variables with parameter $\lambda_n > 0$ such that $\lambda_n = \lambda_{n+T}$, $n \geq 1$.

Then $\{Y_n\}$ a T-periodic sequence, that is, for every choice of integers $i_1 < \cdots < i_n$, $(Y_{i_1}, ..., Y_{i_n})$ and $(Y_{i_1+T}, ..., Y_{i_n+T})$ are identically distributed.

If $\lambda = \min_{1 \le i \le T} \lambda_i$ and C is the number of λ_i's, $i \le T$, equal to λ, then we check directly that $v_n = -\frac{1}{\lambda} \log \frac{AT}{n}$, $n \ge 1$, is a sequence of normalized levels for $\{Y_n\}_{n \ge 1}$ with $\tau = CA$ and for $\{\max(Y_n, Y_{n+1})\}_{n \ge 1}$ with $\tau = 2CA$:

$$\text{(2.2)} \quad \lim_{n\to\infty} \sum_{i=1}^{[na]} P\Big(\max(Y_i, Y_{i+1}) > v_n\Big) = \lim_{n\to\infty} \frac{na}{T} \sum_{i=1}^{T} \Big[1 - P\Big(Y_i \le v_n,\ Y_{i+1} \le v_n\Big)\Big] =$$

$$= \lim_{n\to\infty} \frac{na}{T} \sum_{i=1}^{T} \left[\left(\frac{AT}{n}\right)^{\lambda_i/\lambda} + \left(\frac{AT}{n}\right)^{\lambda_{i+1}/\lambda} - \left(\frac{AT}{n}\right)^{(\lambda_i+\lambda_{i+1})/\lambda}\right] = a\,2\,CA\ .$$

Let us consider the T-periodic sequence

$$X_n = \max(Y_n, Y_{n+1}) + X\ , \qquad n \ge 1\ ,$$

where X is independent of $\{Y_n\}$.

Then $\{X_n^x\}_{n \ge 1}$ is a T-periodic and 2-dependent sequence. Applying (2.2) for $u_n = -\frac{1}{\lambda} \log \frac{1/2T}{n}$, we obtain

$$\lim_{n\to\infty} \sum_{i=1}^{[na]} P(X_i^x > u_n) = \lim_{n\to\infty} P\Big(\max(Y_i, Y_{i+1}) + x > u_n\Big) = a\,C\,e^{\lambda x}\ .$$

Therefore, $u_n = u_n(\tau)$ for $\{X_n^x\}$ with $\tau = C\,e^{\lambda x}$.

Simple calculations enables us to obtain

$$n\,P\Big(X_i^x \le u_n < X_{i+1}^x\Big) \underset{n\to\infty}{\longrightarrow} \nu_i,$$

$$\text{where} \quad \nu_i = \frac{1}{2}\,T\,e^{\lambda x} \ \text{ if } \lambda_{i+2} = \lambda \quad \text{and} \quad \nu_i = 0 \ \text{ for the other cases}\ ;$$

$$\frac{n}{T} \sum_{i=1}^{T} P\Big(X_i^x \le u_n < X_{i+1}^x\Big) \underset{n\to\infty}{\longrightarrow} \nu = \frac{1}{2}\,\tau$$

and

$$\frac{1}{T} \sum_{i=1}^{T} \frac{\nu_i}{\nu} P\Big(X_{i+1}^x > u_n,\ X_{i+2}^x > u_n,\ X_{i+3}^x \le u_n \ \Big|\ X_i^x \le u_n < X_{i+1}^x\Big) \underset{n\to\infty}{\longrightarrow} \Pi(2) = 1\ .$$

Therefore, by applying the results in Ferreira ([5]), we conclude that, for each x, it holds

$$S_n[X_n^x, u_n] \xrightarrow[n\to\infty]{d} S[\tfrac{1}{2}\,C\,e^{\lambda x}, \Pi], \qquad \text{with } \ \Pi(2) = 1\ ,$$

and then $\{S_n[X_n, u_n]\}_{n \ge 1}$ converges, in distribution, to a doubly stochastic compound Poisson process with intensity $\frac{1}{2}\,C\,e^{\lambda X}$ and multiplicity distribution Π. ◻

A more interesting model corresponding to the situations where X and $\{Y_n\}$ are not independent can be treated by using the ideas from the following example.

Example 2.2. Let $X = \{J_n\}_{n\geq 1}$ be the following sequence of discrete random variables:

$$P(J_n = k) = \delta_{n,k}, \quad \text{for } k = 1, ..., m, \qquad \text{with } \sum_{k=1}^{m} \delta_{n,k} = 1, \quad n \geq 1 \ .$$

Consider a sequence $\{X_n\}_{n\geq 1}$ whose finite distributions are determined by $X = \{J_n\}_{n\geq 1}$ as follows:

$$\begin{aligned} P\Big(X_{i_1} \leq x_1,\ ...,\ X_{i_p} \leq x_p \ \Big|\ \{J_n\}_{n\geq 1} = \{j_n\}_{n\geq 1}\Big) &= \prod_{s=1}^{p} P\Big(X_{i_s} \leq x_s \ \Big|\ J_{i_s} = j_{i_s}\Big) \\ &= \prod_{s=1}^{p} F_{j_{i_s}}(x_s) \ , \end{aligned}$$

with non degenerated distributions $F_1, ..., F_m$.

Therefore $\{X_n^{\{j_n\}}\}$ is a sequence of independent and non-identically distributed variables with $P(X_i^{\{j_n\}} \leq x) = F_{j_i}(x)$.

Suppose that, for each $k \in \{1, ..., m\}$,

$$\frac{1}{n} \sum_{i=1}^{n} 1_{\{J_i = k\}} \underset{n\to\infty}{\overset{\text{a.s.}}{\longrightarrow}} N_k \ ,$$

where N_k is a positive random variable, and that

$$n\Big(1 - F_k(u_n)\Big) \underset{n\to\infty}{\longrightarrow} \tau_k > 0 \ .$$

Then, for almost every realization $\{J_n\}_{n\geq 1} = \{j_n\}_{n\geq 1}$, if $\eta_k = \lim_{n\to\infty} \frac{1}{n}\, \eta_{k,n} = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} 1_{\{j_i = k\}}$, for each $k \in \{1, ..., m\}$, it holds

$$\begin{aligned} \sum_{i=1}^{[na]} P\Big(X_i^{\{j_n\}} > u_n\Big) = \sum_{i=1}^{[na]} \frac{\tau_{j_i}}{n} + o(1) = \sum_{k=1}^{m} \frac{\eta_{k,[na]}}{n}\, \tau_k + o(1) \quad \underset{n\to\infty}{\longrightarrow} \\ \underset{n\to\infty}{\longrightarrow} \quad a \sum_{k=1}^{m} \eta_k\, \tau_k = a\, \tau(\{j_n\}_{n\geq 1}) \ . \end{aligned}$$

Therefore, $u_n = u_n(\tau)$ for $\{X_n^{\{j_n\}}\}_{n\geq 1}$.

Since $\{X_n\}$ conditional on $\{J_n\}_{n\geq 1}$ is a sequence of independent variables, then $S[X_n^{\{j_n\}}, u_n]$ converges to an homogeneous Poisson process $S[\tau]$, with intensity τ (see Hüsler ([8])). Therefore, $S_n[X_n, u_n]$ converges to a Cox process with stochastic intensity

$$T = \tau(\{J_n\}) = \sum_{k=1}^{m} N_k\, \tau_k \ . \ \square$$

In the next example $\theta(x)$ and $\Pi^{(x)}$ are not independent of x and we find the limiting multiplicities distributions with the help of results under the local dependence hypotheses.

Example 2.3. Let X be a random variable with values in $\{1, 2\}$ and independent of a sequence $\{Y_n\}$ of i.i.d. variables with continuous common d.f.. Define

$$X_n = \max(Y_n,\, Y_{n-X}, Y_{n-2})\,, \qquad n \geq 1\ .$$

If $u_n = u_n(\tau)$ for $\{Y_n\}$ then $u_n = u_n(3\,\tau)$ for $\{X_n^1\}$ and $u_n = u_n(2\,\tau)$ for $\{X_n^2\}$.

The 2-dependent sequence $\{X_n^1\}$ satisfies the condition $D''(u_n)$ and, by applying the results in Leadbetter and Nandagopalan ([13]), we conclude that $S[X_n^1, u_n] \xrightarrow[n\to\infty]{d} S[\frac{1}{3}\tau, \Pi^{(1)}]$ with $\Pi^{(1)}(3) = 1$.

The 2-dependent sequence $\{X_n^2\}$ does not satisfy the condition $D''(u_n)$ but satisfies the condition $D^{(3)}(u_n)$ from Chernick *et al.* ([2]) and the condition $\widetilde{D}^2(u_n)$ from Ferreira ([6]). It holds

$$\lim_{n\to\infty} nP(X_1^2 > u_n) = 2\,\tau\ ,$$

$$\nu = \lim_{n\to\infty} nP(X_1^2 \leq u_n < X_2^2) = 2\,\tau$$

and

$$\beta = \lim_{n\to\infty} k_n \sum_{j=3}^{[n/k_n]-1} P\Big(X_1^2 \leq u_n < X_2^2,\ X_j^2 \leq u_n < X_{j+1}^2\Big)$$

$$= \lim_{n\to\infty} nP\Big(X_1^2 \leq u_n < X_2^2,\ X_3^2 \leq u_n < X_4^2\Big) = \tau\ ,$$

for any sequence $\{k_n\}$ of positive integers such that $k_n \xrightarrow[n\to\infty]{} \infty$ and $n/k_n \xrightarrow[n\to\infty]{} \infty$.

Then, applying directly the results in Ferreira ([6]) we obtain that $\{X_n^2\}$ has extremal index $\theta = \frac{\nu-\beta}{2\tau} = \frac{1}{2}$ and

$$\Pi^{(2)}(\{1\}) = \lim_{n\to\infty} \frac{n}{\nu-\beta}\; P\Big(X_1^2 \le u_n,\; X_2^2 \le u_n < X_3^2,\; X_4^2 \le u_n,\; X_5^2 \le u_n\Big) = 0\ ;$$

$$\Pi^{(2)}(\{2\}) = \lim_{n\to\infty} \frac{n}{\nu-\beta}\; P\Big(X_1^2 \le u_n < X_2^2,\; X_3^2 \le u_n < X_4^2,\; X_5^2 \le u_n\Big) = 1\ .$$

Therefore $S_n[X_n, u_n]$ converges in distribution to a point process which distribution is a mixture of $S[\frac{1}{3}\tau, \Pi^{(1)}]$ and $S[\frac{1}{2}\tau, \Pi^{(2)}]$ directed by X. □

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