PORTUGALIAE MATHEMATICA
Vol. 61 Fasc. 1 – 2004
Nova Série

BOUNDED HOLOMORPHIC MAPPINGS AND THE COMPACT APPROXIMATION PROPERTY IN BANACH SPACES

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Abstract: We study the compact approximation property in connection with the space of bounded holomorphic mappings on a Banach space. When U is a bounded balanced open subset of a Banach space E, we show that the predual of the space of the bounded holomorphic functions on U, $G^\infty(U)$, has the compact approximation property if and only if E has the compact approximation property. We also show that E has the compact approximation property if and only if each continuous Banach-valued polynomial on E can be uniformly approximated on compact sets by polynomials which are weakly continuous on bounded sets.

1 – Introduction

Let E and F be complex Banach spaces, and let $L(E;F)$ be the Banach space of all continuous linear operators $T\colon E \to F$. E is said to have the approximation property (AP for short) if given a compact set $K \subset E$ and $\epsilon > 0$, there is a finite rank operator $T \in L(E;E)$ such that $\|Tx - x\| < \epsilon$ for every $x \in K$. E is said to have the compact approximation property (CAP for short) if given a compact set $K \subset E$ and $\epsilon > 0$, there is a compact operator $T \in L(E;E)$ such that $\|Tx - x\| < \epsilon$ for every $x \in K$. The AP implies the CAP, but Willis [12] has shown that the reverse implication is not true.

Let U be an open subset of E, and let $\mathcal{H}^\infty(U;F)$ denote the Banach space of all bounded holomorphic mappings $f\colon U \to F$, with the norm of the supremum.

Received: December 2, 2002.
AMS Subject Classification: 46G20, 46B28, 46G25.
Keywords: Banach spaces; compact approximation property; bounded holomorphic functions.

When $F = \mathbb{C}$, we write $\mathcal{H}^\infty(U)$ instead of $\mathcal{H}^\infty(U;\mathbb{C})$. Let $G^\infty(U)$ denote the predual of $\mathcal{H}^\infty(U)$ constructed by Mujica [8]. If U is open, balanced and bounded, then Mujica [8] proved that E has the AP if and only if $G^\infty(U)$ has the AP if and only if, for each Banach space F, every $f \in \mathcal{H}^\infty(U;F)$ lies in the τ_γ-closure of the subspace of all $g \in \mathcal{H}^\infty(U;F)$ with a finite dimensional range, where τ_γ is a locally convex topology on $\mathcal{H}^\infty(U;F)$ which is finer than the compact-open topology.

In this paper we show that if U is open, balanced and bounded, then E has the CAP if and only if $G^\infty(U)$ has the CAP if and only if, for each Banach space F, every $f \in \mathcal{H}^\infty(U;F)$ lies in the τ_γ-closure of the subspace of all $g \in \mathcal{H}^\infty(U;F)$ with a relatively compact range. We obtain this result by combining the techniques of Mujica [8] and results of Aron and Prolla [2] and Aron, Hervés and Valdivia [1].

We also show that E has the CAP if and only if each continuous Banach-valued polynomial on E can be uniformly approximated on compact sets by compact polynomials, or equivalently, by polynomials which are weakly continuous on bounded sets. This improves results of Mujica and Valdivia [10, Proposition 2.2] and Mujica [9, Proposition 3.3].

2 – The compact approximation property

The symbol $\mathbb{C}$ represents the field of all complex numbers, $\mathbb{N}$ represents the set of all positive integers, and $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$.

An operator T in $L(E;F)$ is said to have a finite rank if $T(E)$ is finite dimensional, and an operator T in $L(E;F)$ is called a compact operator if T takes bounded subsets of E to relatively compact subsets of F. Let $L_k(E;F)$ denote the subspace of all compact operators of $L(E;F)$. When $F = \mathbb{C}$ we write E' instead of $L(E;\mathbb{C})$.

The following characterization of the CAP is similar to the characterization of the AP due to Grothendieck (see [6, Theorem 1.e.4]). τ_c will always denote the compact-open topology.

Proposition 1. *For a Banach space E the following statements are equivalent:*

(i) *E has the CAP.*

(ii) $L(E;E) = \overline{L_k(E;E)}^{\tau_c}$.

(iii) *For every Banach space F,* $L(F;E) = \overline{L_k(F;E)}^{\tau_c}$.

(**iv**) *For every Banach space* F, $L(E;F) = \overline{L_k(E;F)}^{\tau_c}$.

(**v**) *For every choice of* $(x_n)_{n=1}^{\infty} \subset E$ *and* $(x'_n)_{n=1}^{\infty} \subset E'$ *such that* $\sum_{n=1}^{\infty} \|x_n\|.\|x'_n\| < \infty$ *and* $\sum_{n=1}^{\infty} x'_n(Tx_n) = 0$ *for every* $T \in L_k(E;E)$, *we have that* $\sum_{n=1}^{\infty} x'_n(x_n) = 0$. ∎

Using Proposition 1 (v) we easily get the following

Corollary 2. *If* E *is a reflexive Banach space, then* E' *has the CAP if and only if* E *has the CAP.* ∎

It is known that a Banach space E has the AP if E' has the AP (see [6, Theorem 1.e.7]). But the following problem, mentioned by Casazza [3, Problem 8.5], still remains open.

Problem: Let E be a Banach space. If E' has the CAP, must E have the CAP? □

Corollary 2 is an affirmative answer in the case of reflexive Banach spaces.

It is easy to show that if E has the CAP, then every complemented subspace of E has the CAP. Using Proposition 1 (v) and [5, Proposition 1] we have the following

Proposition 3. *For a reflexive Banach space* E, *the following statements are equivalent:*

(**a**) *E has the CAP.*

(**b**) *Every complemented subspace of E has the CAP.*

(**c**) *Every complemented and separable subspace of E has the CAP.* ∎

3 – The compact approximation property and bounded holomorphic mappings

The letter U denotes a nonvoid open subset of E. The symbol U_E represents the unit open ball of E, and the symbol B_E represents the closed unit ball of E.

Let $\mathcal{P}(E;F)$ denote the vector space of all continuous polynomials from E into F. We say that $P \in \mathcal{P}(E;F)$ is compact if P takes bounded subsets of E to relatively compact subsets of F. Let $\mathcal{P}_k(E;F)$ denote the subspace of all compact members of $\mathcal{P}(E;F)$.

Let $\mathcal{P}_w(E;F)$ (resp. $\mathcal{P}_{wu}(E;F)$) denote the subspace of all members of $\mathcal{P}(E;F)$ which are weakly (resp. weakly uniformly) continuous in the bounded subsets of E.

Let $\mathcal{P}(^mE;F)$ denote the subspace of all m-homogeneous members of $\mathcal{P}(E;F)$, let $\mathcal{P}_w(^mE;F)$ (resp. $\mathcal{P}_{wu}(^mE;F)$) denote the subspace of all members of $\mathcal{P}(^mE;F)$ which are weakly (resp. weakly uniformly) continuous on bounded subsets of E, for every $m \in \mathbb{N}_0$.

Let $\mathcal{H}(U;F)$ denote the vector space of all holomorphic mappings from U into F. Let $\mathcal{H}^\infty(U;F)$ denote the subspace of all bounded members of $\mathcal{H}(U;F)$, and let $\mathcal{H}^\infty_K(U;F)$ be the subspace of all members of $\mathcal{H}^\infty(U;F)$ which have relatively compact range.

When $F=\mathbb{C}$ we write $\mathcal{H}^\infty(U)$ and $\mathcal{P}(^mE)$ instead of $\mathcal{H}^\infty(U;\mathbb{C})$ and $\mathcal{P}(^mE;\mathbb{C})$.

We refer to [4] or [7] for the properties of $\mathcal{P}(E;F)$ and $\mathcal{H}(U;F)$, and to [1] and [2] for the properties of $\mathcal{P}_w(E;F)$ and $\mathcal{P}_{wu}(E;F)$.

The symbol Λ denotes a directed set.

The following result of J.Mujica [7] is essential to prove Theorem 5.

Theorem 4 ([8, Theorem 2.1])**.** *Let U be an open subset of a Banach space E. Then there is a Banach space $G^\infty(U)$ and a mapping $\delta_U \in \mathcal{H}^\infty(U;G^\infty(U))$ with $\|\delta_U\| = 1$ and with the following universal property: For each Banach space F and each mapping $f \in \mathcal{H}^\infty(U;F)$, there is a unique operator $T_f \in L(G^\infty(U);F)$ such that $T_f \circ \delta_U = f$. The correspondence*

$$f \in \mathcal{H}^\infty(U;F) \longrightarrow T_f \in L(G^\infty(U);F)$$

is an isometric isomorphism. These properties characterize $G^\infty(U)$ uniquely up to an isometric isomorphism. ∎

The space $G^\infty(U)$ is defined as the closed subspace of all linear functionals $u \in \mathcal{H}^\infty(U)'$ such that $u|_{B_{\mathcal{H}^\infty(U)}}$ is τ_c-continuous, and the evaluation mapping $\delta_U\colon x \in U \to \delta_x \in G^\infty(U)$ is defined by $\delta_x\colon f \in \mathcal{H}^\infty(U) \to f(x) \in \mathbb{C}$, for every $x \in U$.

Definition ([8, Theorem 4.8])**.** Let E and F be Banach spaces, and let U be an open subset of E. Let τ_γ denote the locally convex topology on $\mathcal{H}^\infty(U;F)$ generated by all the seminorms of the form

$$p(f) = \sup_j \alpha_j \|f(x_j)\| ,$$

where $(x_j)_{j=1}^\infty$ varies over all sequences in U, and $(\alpha_j)_{j=1}^\infty$ varies over all sequences of positive numbers tending to zero. □

The next result is similar to a result of J.Mujica [8, Theorem 5.6].

Theorem 5. *Let E be a Banach space, and let U be a balanced, bounded, and open set in E. The following statements are equivalent:*

(**a**) *E has the CAP.*

(**b**) *For each Banach space F, $\mathcal{H}^\infty(U;F) = \overline{\mathcal{P}_w(E;F)}^{\tau_\gamma}$.*

(**c**) *For each Banach space F, $\mathcal{H}^\infty(U;F) = \overline{\mathcal{P}_k(E;F)}^{\tau_\gamma}$.*

(**d**) *For each Banach space F, $\mathcal{H}^\infty(U;F) = \overline{\mathcal{H}^\infty_K(U;F)}^{\tau_\gamma}$.*

(**e**) *$\delta_U \in \overline{\mathcal{H}^\infty_K(U;G^\infty(U))}^{\tau_\gamma}$.*

(**f**) *$G^\infty(U)$ has the CAP.*

(**g**) *For each Banach space F, and for each open set $V \subset F$,*
$$\mathcal{H}^\infty(V;E) = \overline{\mathcal{H}^\infty_K(V;E)}^{\tau_\gamma}.$$

(**h**) *$I_U \in \overline{\mathcal{H}^\infty_K(U;E)}^{\tau_\gamma}$.*

Proof: (**a**) $\Rightarrow$ (**b**): Let $f \in \mathcal{H}^\infty(U;F)$. Let p be a continuous seminorm on $(\mathcal{H}^\infty(U;F), \tau_\gamma)$. Then by [8, Proposition 5.2] there is a $P \in \mathcal{P}(E;F)$ such that $p(P - f) < \frac{\epsilon}{2}$. On the other hand, by [8, Proposition 4.9] $\tau_\gamma = \tau_c$ on $\mathcal{P}(^kE;F)$, for every $k \in \mathbb{N}_0$. Now let $P = P_0 + P_1 + \cdots + P_n$, where $P_j \in \mathcal{P}(^jE;F)$, for every $j = 0, 1, ..., n$. Hence, since E has the CAP, then by [10, Proposition 2.1], and [9, Proposition 3.3] there is a $Q_j \in \mathcal{P}_w(^jE;F)$ such that $p(Q_j - P_j) < \frac{\epsilon}{2(n+1)}$, for every $j = 0, 1, ..., n$. Note that $Q_0 + Q_1 + \cdots + Q_n = Q \in \mathcal{P}_w(E;F)$. Since $p(Q - P) < \frac{\epsilon}{2}$, then $p(Q - f) \leq p(Q - P) + p(P - f) < \epsilon$. Thus, we have (b).

(**b**) $\Rightarrow$ (**c**): By [1, Theorem 2.9] we have $\mathcal{P}_w(E;F) = \mathcal{P}_{wu}(E;F)$, and by [2, Lemma 2.2] we have $\mathcal{P}_{wu}(E;F) \subset \mathcal{P}_k(E;F)$. Hence from (b) we get (c).

(**c**) $\Rightarrow$ (**d**): Since U is bounded, we have $\mathcal{P}_k(E;F) \subset \mathcal{H}^\infty_K(U;F)$. Hence from (c) we get (d).

(**d**) $\Rightarrow$ (**e**): We know from Theorem 4 that $\delta_U \in \mathcal{H}^\infty(U;G^\infty(U))$. But, taking $F = G^\infty(U)$ in (d), we have that $\mathcal{H}^\infty(U;G^\infty(U) = \overline{\mathcal{H}^\infty_K(U;G^\infty(U))}^{\tau_\gamma}$.

(**e**) $\Rightarrow$ (**f**): Let $\delta_U \in \overline{\mathcal{H}^\infty_K(U;G^\infty(U))}^{\tau_\gamma}$. It is enough to show that the identity mapping I on $G^\infty(U)$ belongs to $\overline{L_k(G^\infty(U);G^\infty(U))}^{\tau_c}$. In fact, from (e), there is a net $(f_\alpha)_{\alpha\in\Lambda} \subset \mathcal{H}^\infty_K(U;G^\infty(U))$ such that $f_\alpha \xrightarrow{\tau_\gamma} \delta_U$. Then by Theorem 4, and [8, Proposition 3.4 and Theorem 4.8] we have a corresponding net $(T_{f_\alpha})_{\alpha\in\Lambda} \subset L_k(G^\infty(U);G^\infty(U))$ which converges to I for τ_c. Therefore, we have $I \in \overline{L_k(G^\infty(U);G^\infty(U))}^{\tau_c}$.

(**f**) $\Rightarrow$ (**a**): Since by [8, Proposition 2.3] E is topologically isomorphic to a complemented subspace of $G^\infty(U)$, it follows from (f) that E has the CAP.

(**a**) $\Rightarrow$ (**g**): Suppose that E has the CAP. Let F be a Banach space, and let $V \subset F$ be an open subset. Let $f \in \mathcal{H}^\infty(V;E)$. Then, by Theorem 4, there is a $T_f \in L(G^\infty(V);E)$. Hence, by Proposition 1, there is a net $(T_\alpha)_{\alpha\in\Lambda} \subset L_k(G^\infty(V);E)$ such that $T_\alpha \xrightarrow{\tau_c} T_f$. By Theorem 4 $(T_\alpha)_{\alpha\in\Lambda} = (T_{f_\alpha})_{\alpha\in\Lambda}$ correspond to a net $(f_\alpha)_{\alpha\in\Lambda}$ in $\mathcal{H}^\infty(V;E)$. By [8, Proposition 3.4], $(f_\alpha)_{\alpha\in\Lambda} \subset \mathcal{H}_K^\infty(V;E)$, and by [8, Theorem 4.8], $f_\alpha \xrightarrow{\tau_\gamma} f$. Hence we have (g).

(**g**) $\Rightarrow$ (**a**): By (g) $\mathcal{H}^\infty(U_F;E) = \overline{\mathcal{H}_K^\infty(U_F;E)}^{\tau_\gamma}$. We claim that $L(G^\infty(U_F);E) = \overline{L_k(G^\infty(U_F);E)}^{\tau_c}$. Let $T \in L(G^\infty(U_F);E)$. Then by Theorem 4 there is a $f \in \mathcal{H}^\infty(U_F;E)$ such that $T = T_f$. Hence, by hypothesis there existe a net $(f_\alpha)_{\alpha\in\Lambda} \subset \mathcal{H}_K^\infty(U_F;E)$ such that $f_\alpha \xrightarrow{\tau_\gamma} f$. By Theorem 4 and [8, Proposition 3.4, and Theorem 4.8] there is a corresponding net $(T_{f_\alpha})_{\alpha\in\Lambda} \subset L_k(G^\infty(U_F);E)$ which converges to T for τ_c. Hence we have that $L(G^\infty(U_F);E) = \overline{L_k(G^\infty(U_F);E)}^{\tau_c}$. We claim that $L(F;E) = \overline{L_k(F;E)}^{\tau_c}$. Let $A \in L(F;E)$. By [8, Proposition 2.3], there are operators $S \in L(F;G^\infty(U_F))$ and $R \in L(G^\infty(U_F);F)$ such that $R \circ S(y) = y$ for every $y \in F$. Then, $A \circ R \in L(G^\infty(U_F);E)$ and hence there is a net $(B_\alpha)_{\alpha\in\Lambda} \subset L_k(G^\infty(U_F);E)$ which converges to $A \circ R$ for τ_c. Thus, $B_\alpha \circ S \in L_k(F;E)$ and converges to $A \circ R \circ S = A$ for τ_c. Therefore we have that $L(F;E) = \overline{L_k(F;E)}^{\tau_c}$.

(**d**) $\Rightarrow$ (**h**): Obvious.

(**h**) $\Rightarrow$ (**d**): Suppose that $I_U \in \overline{\mathcal{H}_K^\infty(U;E)}^{\tau_\gamma}$. Let $f \in \mathcal{H}^\infty(U;F)$, let p be a continuous seminorm on $(\mathcal{H}^\infty(U;F),\tau_\gamma)$. We want to find $g \in \mathcal{H}_K^\infty(U;F)$ such that $p(g-f) < 1$. We may assume that

$$p(h) = \sup_j \alpha_j \|h(x_j)\|\,, \qquad \forall\, h \in \mathcal{H}^\infty(U;F)\ ,$$

where $(x_j)_{j=1}^\infty \subset U$ and $(\alpha_j)_{j=1}^\infty \in c_0$ with $\alpha_j > 0$ for every $j \in \mathbb{N}$. By [8, Proposition 5.2] there exists $P \in \mathcal{P}(E;F)$ such that

$$p(P-f) < \frac{1}{2}\ .$$

Write $P = P_0 + P_1 + \cdots + P_m$, with $P_k \in \mathcal{P}(^kE;F)$, $\forall\, k = 0,1,...,m$. Certainly $P_0 \in \mathcal{H}_K^\infty(U;F)$. For every $k = 1,...,m$ we shall find $u_k \in \mathcal{H}_K^\infty(U;E)$ such that

$$(*) \qquad p(P_k \circ u_k - P_k) < \frac{1}{2\,m}\ .$$

Then it will follow that

$$P_0+\sum_{k=1}^{m} P_k\circ u_k \ \in\ \mathcal{H}^\infty_K(U;F)$$

and

$$p\Big(P_0+\sum_{k=1}^{m} P_k\circ u_k - f\Big) \,=\, p\Big(P-P_1-P_2-\cdots-P_m+\sum_{k=1}^{m} P_k\circ u_k - f\Big) \,<\, 1\ ,$$

thus proving (d).

Now, fix k with $1\le k\le m$, let $\beta_j=\sqrt[k]{\alpha_j}$, for every $j\in\mathbb{N}$ and let $K=\{\beta_j\,x_j\colon\, j\in\mathbb{N}\}\cup\{0\}$. Since K is compact, there exists $\delta>0$ such that $\|P_k(y)-P_k(x)\|<\frac{1}{2\,m}$ whenever $x\in K$ and $\|y-x\|<\delta$. By (h), there exists $u_k\in\mathcal{H}^\infty_K(U;E)$ such that $\sup\limits_j \beta_j\|u_k(x_j)-x_j\|<\delta$. Hence $p(P_k\circ u_k-P_k)=\sup\limits_j\|P_k(\beta_j u_k(x_j))-P_k(\beta_j x_j)\|<\frac{1}{2\,m}$, showing that u_k satisfies $(*)$. Thus the proof of the theorem is complete. ∎

Observe that in the previous theorem, in item (g), taking the weaker condition $\mathcal{H}^\infty(U_E;E)=\overline{\mathcal{H}^\infty_K(U_E;E)}^{\,\tau_\gamma}$ we can obtain the same result.

Using the same proof of [10, Proposition 2.2], we can also prove the following

Proposition 6. *Let E and F be Banach spaces. The following statements are equivalent:*

(**a**) $\mathcal{P}(E;F)=\overline{\mathcal{P}_k(E;F)}^{\,\tau_c}$.

(**b**) $\mathcal{P}(^mE;F)=\overline{\mathcal{P}_k(^mE;F)}^{\,\tau_c}$ *for every* $m\in\mathbb{N}$. ∎

In the proof of the next Corollary we will use the following version of a theorem of Ryan [11], which appeared in [9] (see also [8, Theorems 2.4 and 4.1]): For each Banach space E and each $m\in\mathbb{N}$ let $Q(^mE)$ be the closed subspace of all linear continuous functionals $v\in\mathcal{P}(^mE)'$ such that $v|_{B_{\mathcal{P}(^mE)}}$ is τ_c-continuous, and let $\delta_m\colon\, x\in E\to\delta_x\in Q(^mE)$ denote the evaluation mapping, that is, $\delta_x(P)=P(x)$, for every $x\in E$ and $P\in\mathcal{P}(^mE)$. Then $Q(^mE)$ is a Banach space with the norm induced by $\mathcal{P}(^mE)'$, and $\delta_m\in\mathcal{P}(^mE;Q(^mE))$ with $\|\delta_m\|=1$. The pair $(Q(^mE),\delta_m)$ has the following universal property: For each Banach space F and each $P\in\mathcal{P}(^mE;F)$, there is a unique operator $T_P\in L(Q(^mE);F)$ such that $T_P\circ\delta_m=P$. The correspondence

$$P\in\mathcal{P}(^mE;F)\ \longrightarrow\ T_P\in L(Q(^mE);F)$$

is an isometric isomorphism, and is also a topological isomorphism when both spaces are endowed with the compact-open topology τ_c. Moreover $P \in \mathcal{P}_k({}^mE;F)$ if, and only if $T_P \in L_k(Q({}^mE);F)$.

Corollary 7. *For a Banach space E, the following statements are equivalent:*

(**a**) *E has the CAP.*

(**b**) *For each Banach space F, $\mathcal{P}(E;F) = \overline{\mathcal{P}_w(E;F)}^{\tau_c}$.*

(**c**) *For each Banach space F, $\mathcal{P}(E;F) = \overline{\mathcal{P}_k(E;F)}^{\tau_c}$.*

(**d**) *$Q({}^mE)$ has the CAP, for every $m \in \mathbb{N}$.*

Proof: (**a**) $\Rightarrow$ (**b**): It follows from Theorem 5 and the fact that $\tau_\gamma \geq \tau_c$ on $\mathcal{H}^\infty(U;F)$ (see [8, Proposition 4.9]).

(**b**) $\Rightarrow$ (**c**): Clear.

(**c**) $\Rightarrow$ (**d**): It follows from Proposition 6 and the aforementioned result of Ryan that $L(Q({}^mE);F) = \overline{L_k(Q({}^mE);F)}^{\tau_c}$ for each Banach space F. Hence by Proposition 1 $Q({}^mE)$ has the CAP, for every $m \in \mathbb{N}$.

(**d**) $\Rightarrow$ (**a**): Clear since $Q({}^1E) = E$. ∎

Corollary 7 improves [10, Proposition 2.2] and [9, Proposition 3.3].

Willis [12] has constructed a Banach space Z which has the CAP, but does not have the AP. If U is an open, balanced, bounded subset of Z, then it follows from Theorem 5 and [8, Theorem 5.4] that $G^\infty(U)$ has the CAP, but does not have the AP. The same is true for $Q({}^mZ)$ for every $m \in \mathbb{N}$.

One can obtain results similar to Theorem 5 and Corollary 7 for the metric compact approximation property. For the definition see [3].

ACKNOWLEDGEMENTS – This paper is based on a part of the author's doctoral thesis at the Universidade Estadual de Campinas, Brazil, written under supervision of Professor Jorge Mujica. The author would like to thank his thesis advisor.

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