Central Extension of the Yangian Double

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Abstract

A central extension $\widehat{DY(\mathfrak{g})}$ of the double of the Yangian is defined for a simple Lie algebra $\mathfrak{g}$ with complete proof for $\mathfrak{g} = \mathfrak{sl}_2$. Basic representations and intertwining operators are constructed for $\widehat{DY(\mathfrak{sl}_2)}$.

Résumé

On définit une extension centrale $\widehat{DY(\mathfrak{g})}$ du double du Yangien pour une algèbre de Lie simple $\mathfrak{g}$ avec des preuves complètes pour le cas $\mathfrak{g} = \mathfrak{sl}_2$. On construit des représentations de base et des opérateurs d'entrelacement pour $\widehat{DY(\mathfrak{sl}_2)}$.

1 Introduction

The Yangian $Y(\mathbf{g})$ was introduced by V. Drinfeld [D1] as a Hopf algebra quantizing the rational solution $r(u) = \frac{e_i \otimes e^i}{u}$ of classical Yang-Baxter equation. As a Hopf algebra $Y(g)$ is a deformation of the universal enveloping algebra $U(\mathbf{g}[u])$ of polynomial currents to a simple Lie algebra $\mathbf{g}$ with respect to cobracket defined by $r(u)$. Unfortunately, up to the moment the representation theory of the Yangian is not so rich in applications as it takes place, for instance, for quantum affine algebras [JM]. We could mention the following gaps:

(i) The Yangian is not quasitriangular Hopf algebra, but pseudoquasitriangular Hopf algebra [D1];

(ii) There are no nontrivial examples of infinite-dimensional representations of $Y(\mathbf{g})$.

In order to get quasitriangular Hopf algebra one should introduce the quantum double $DY(\mathbf{g})$ of the Yangian. Detailed analisys of $DY(\mathbf{g})$ was done in [KT] together with explicit description of the universal R-matrix for $DY(\mathbf{g})$ (complete for $DY(sl_2)$ and partial in general case).

The most important examples of infinite dimensional representations of (quantum) affine algebras appear for nonzero value of central charge. Analogously, in the

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case of the Yangian we could expect the appearance of infinite dimensional representations only after central extension of $DY(\mathbf{g})$. This program is realized in this paper with complete proof for $DY(sl_2)$: we give a description of central extension $\widehat{DY(sl_2)}$ and construct its basic representations in bosonized form. In general case we present a description of $\widehat{DY(\mathbf{g})}$ without complete proof. In the forthcoming paper [KLP] we demonstrate, following general scheme of [DFJMN], how our construction produce the formulas for correlation functions in rational models [S].

The central extension of $DY(\mathbf{g})$ could be constructed in two ways. In Faddev--Reshetikhin-Takhtajan approach [FRT] we could describe $DY(\mathbf{g})$ by the set of equations

$$R^{\pm}_{V,W}(u-v)L^{\pm}_{V}(u)L^{\pm}_{W}(v) = L^{\pm}_{W}(v)L^{\pm}_{V}(u)R^{\pm}_{V,W}(u-v), \tag{1}$$

$$R^{+}_{V,W}(u-v)L^{+}_{V}(u)L^{-}_{W}(v) = L^{-}_{W}(v)L^{+}_{V}(u)R^{+}_{V,W}(u-v) \tag{2}$$

for matrix valued generating functions $L^{\pm}_{V}(u)$, $L^{\pm}_{W}(v)$ of $DY(\mathbf{g})$, where $V(u), W(v)$ are finite dimensional representations of $DY(\mathbf{g})$, $R^{\pm}_{V,W}$ are images of the universal R-matrix $\mathcal{R}$ and $(\mathcal{R}^{-1})^{21}$. Then, following [RS], we can make a shift of a spectral parameter in R-matrix in equation (2) by a central element:

$$R^{+}_{V,W}(u-v+c\hbar)L^{+}_{V}(u)L^{-}_{W}(v) = L^{-}_{W}(v)L^{+}_{V}(u)R^{+}_{V,W}(u-v). \tag{3}$$

For the construction of representations we should then extract Drinfeld generators of $\widehat{DY(\mathbf{g})}$ from L-operators (3) via their Gauss decomposition [DF].

We prefer another way, originaly used by V.Drinfeld [D2] for his "current" description of quantum affine algebras. The properties of comultiplication for $Y(\mathbf{g})$ show that one can extend the Yangian $Y(\mathbf{g})$ (or its dual with opposite comultiplication $Y^0(\mathbf{g})$) to a new Hopf algebra, adding the derivative d of a spectral parameter u. Alternativly, one can extend $Y(\mathbf{g})$ by automorphisms of shifts of u. The double of this extension is exactly what we want to find. Central element c is dual to derivative d.

The plan of the paper is as follows. First we remind the description of the Yangian $Y(sl_2)$ and of its quantum double from [KT]. Then we construct the central extension $\widehat{DY(sl_2)}$, describe the structure of the universal R-matrix for $\widehat{DY(sl_2)}$ and translate our description into L- operator language. In section 5 we construct basic representation of $\widehat{DY(sl_2)}$ and in the last section we describe the structure of $\widehat{DY(\mathbf{g})}$ in general case (without a proof).

2 $Y(sl_2)$ and its Quantum Double

The Yangian $Y(sl_2)$ can be described as a Hopf algebra generated by the elements e_k, h_k, f_k, $k \geq 0$ subjected to the relations

$$\begin{gathered}
[h_k, h_l] = 0 \ , \qquad [e_k, f_l] = h_{k+l} \ , \\
[h_0, e_l] = 2e_l \ , \qquad [h_0, f_l] = -2f_l \ , \\
[h_{k+1}, e_l] - [h_k, e_{l+1}] = \hbar\{h_k, e_l\} \ , \\
[h_{k+1}, f_l] - [h_k, f_{l+1}] = -\hbar\{h_k, f_l\} \ , \\
[e_{k+1}, e_l] - [e_k, e_{l+1}] = \hbar\{e_k, e_l\} \ , \\
[f_{k+1}, f_l] - [f_k, f_{l+1}] = -h\{f_k, f_l\} \ ,
\end{gathered} \tag{4}$$

where $\hbar$ is a parameter of the deformation, $\{a, b\} = ab + ba$. The comultiplication and the antipode are uniquely defined by the relations

$$\Delta(e_0) = e_0 \otimes 1 + 1 \otimes e_0, \qquad \Delta(h_0) = h_0 \otimes 1 + 1 \otimes h_0, \qquad \Delta(f_0) = f_0 \otimes 1 + 1 \otimes f_0,$$

$$\Delta(e_1) = e_1 \otimes 1 + 1 \otimes e_1 + \hbar h_0 \otimes e_0, \qquad \Delta(f_1) = f_1 \otimes 1 + 1 \otimes f_1 + \hbar f_0 \otimes h_0,$$

$$\Delta(h_1) = h_1 \otimes 1 + 1 \otimes h_1 + \hbar h_0 \otimes h_0 - 2\hbar f_0 \otimes e_0. \tag{5}$$

In terms of generating functions

$$e^+(u) := \sum_{k\geq 0} e_k u^{-k-1}, \qquad f^+(u) := \sum_{k\geq 0} f_k u^{-k-1}, \qquad h^+(u) := 1 + \hbar \sum_{k\geq 0} h_k u^{-k-1}$$

the relations (4) look as follows

$$\begin{gathered}
[h^+(u), h^+(v)] = 0 \ , \\
[e^+(u), f^+(v)] = -\frac{1}{\hbar}\frac{h^+(u) - h^+(v)}{u - v} \ , \\
[h^+(u), e^+(v)] = -\hbar\frac{\{h^+(u), (e^+(u) - e^+(v))\}}{u - v} \ , \\
[h^+(u), f^+(v)] = \hbar\frac{\{h^+(u), (f^+(u) - f^+(v))\}}{u - v} \ , \\
[e^+(u), e^+(v)] = -\hbar\frac{(e^+(u) - e^+(v))^2}{u - v} \ , \\
[f^+(u), f^+(v)] = \hbar\frac{(f^+(u) - f^+(v))^2}{u - v} \ .
\end{gathered} \tag{6}$$

The comultiplication is given by Molev's formulas [M], see also [KT]:

$$\Delta(e^+(u)) = e^+(u) \otimes 1 + \sum_{k=0}^{\infty} (-1)^k \hbar^{2k} \big(f^+(u+\hbar)\big)^k h^+(u) \otimes \big(e^+(u)\big)^{k+1}, \tag{7}$$

$$\Delta(f^+(u)) = 1\otimes f^+(u) + \sum_{k=0}^{\infty}(-1)^k\hbar^{2k}\big(f^+(u)\big)^{k+1}\otimes h^+(u)\big(e^+(u+\hbar)\big)^k \tag{8}$$

$$\Delta(h^+(u)) = \sum_{k=0}^{\infty}(-1)^k(k+1)\hbar^{2k}\big(f^+(u+\hbar)\big)^k h^+(u)\otimes h^+(u)\big(e^+(u+\hbar)\big)^k \tag{9}$$

Let now C be an algebra generated by the elements e_k, f_k, h_k, $(k\in\mathbf{Z})$, with relations (4). Algebra C admits $\mathbf{Z}$-filtration

$$\ldots\subset C_{-n}\subset\ldots\subset C_{-1}\subset C_0\subset C_1\ldots\subset C_n\ldots\subset C \tag{10}$$

defined by the conditions $\deg e_k = \deg f_k = \deg h_k = k$; $\deg x\in C_m\le m$. Let $\bar{C}$ be the corresponding formal completion of C. It is proved in [KT] that $DY(sl_2)$ is isomorphic to $\bar{C}$ as an algebra. In terms of generating functions

$$e^{\pm}(u) := \pm\sum_{\substack{k\ge 0\\ k<0}} e_k u^{-k-1},\qquad f^{\pm}(u) := \pm\sum_{\substack{k\ge 0\\ k<0}} f_k u^{-k-1},\qquad h^{\pm}(u) := 1\pm\hbar\sum_{\substack{k\ge 0\\ k<0}} h_k u^{-k-1},$$
$$e(u) = e^+(u) - e^-(u),\qquad f(u) = f^+(u) - f^-(u)$$

the defining relations for $DY(sl_2)$ look as follows:

$$\begin{aligned}
h^a(u)h^b(v) &= h^b(v)h^a(u),\qquad a,b=\pm,\\
e(u)e(v) &= \frac{u-v+\hbar}{u-v-\hbar}\,e(v)e(u)\\
f(u)f(v) &= \frac{u-v-\hbar}{u-v+\hbar}\,f(v)f(u)\\
h^{\pm}(u)e(v) &= \frac{u-v+\hbar}{u-v-\hbar}\,e(v)h^{\pm}(u)\\
h^{\pm}(u)f(v) &= \frac{u-v-\hbar}{u-v+\hbar}\,f(v)h^{\pm}(u)\\
[e(u),f(v)] &= \frac{1}{\hbar}\delta(u-v)\big(h^+(u)-h^-(v)\big)
\end{aligned} \tag{11}$$

Here

$$\delta(u-v) = \sum_{n+m=-1} u^n v^m$$

satisfies the property

$$\delta(u-v)f(u) = \delta(u-v)f(v).$$

The comultiplication is given by the same formulas

$$\Delta(e^{\pm}(u)) = e^{\pm}(u)\otimes 1 + \sum_{k=0}^{\infty}(-1)^k\hbar^{2k}\big(f^{\pm}(u+\hbar)\big)^k h^{\pm}(u)\otimes\big(e^{\pm}(u)\big)^{k+1},$$

$$\Delta(f^{\pm}(u)) = 1\otimes f^{\pm}(u) + \sum_{k=0}^{\infty}(-1)^k\hbar^{2k}\big(f^{\pm}(u)\big)^{k+1}\otimes h^{\pm}(u)\big(e^{\pm}(u+\hbar)\big)^k$$

$$\Delta(h^{\pm}(u)) = \sum_{k=0}^{\infty}(-1)^k(k+1)\hbar^{2k}\big(f^{\pm}(u+\hbar)\big)^k h^{\pm}(u)\otimes h^{\pm}(u)\big(e^{\pm}(u+\hbar)\big)^k. \tag{12}$$

Subalgebra $Y^+ = Y(sl_2)\subset DY(sl_2)$ is generated by the components of $e^+(u)$, $f^+(u)$, $h^+(u)$ and its dual with opposite comultiplication $Y^- = (Y(sl_2))^0$ is a formal completion (see (10)) of subalgebra, generated by the components of $e^-(u)$, $f^-(u)$, $h^-(u)$.

Let us describe the Hopf pairing of Y^+ and Y^- [KT]. Note that by a Hopf pairing $<,>: A\otimes B\to \mathbf{C}$ we mean bilinear map satisfying the conditions

$$< a, b_1b_2 >=< \Delta(a), b_1\otimes b_2 >,\qquad < a_1a_2, b >=< a_2\otimes a_1, \Delta(b) > \tag{13}$$

The last condition is unusual but convenient in a work with quantum double.

Let $E^{\pm}$, $H^{\pm}$, $F^{\pm}$ be subalgebras (or their completions in Y^- case) generated by the components of $e^{\pm}(u)$, $h^{\pm}(u)$, $f^{\pm}(u)$. Subalgebras $E^{\pm}$ and $F^{\pm}$ do not contain the unit.

The first property of the Hopf pairing $Y^+\otimes Y^-\to\mathbf{C}$ is that it preserves the decompositions

$$Y^+ = E^+H^+F^+,\qquad Y^- = F^-H^-E^-.$$

It means that

$$< e^+h^+f^+, f^-h^-e^- >=< e^+, f^- >< h^+, h^- >< f^+, e^- > \tag{14}$$

for any elements $e^{\pm}\in E^{\pm}$, $h^{\pm}\in H^{\pm}$, $f^{\pm}\in F^{\pm}$. This property defines the pairing uniquely together with the relations

$$< e^+(u), f^-(v) >= \frac{1}{\hbar(u-v)},\qquad < f^+(u), e^-(v) >= \frac{1}{\hbar(u-v)},$$

$$< h^+(u), h^-(v) >= \frac{u-v+\hbar}{u-v-\hbar}.$$

The full information of the pairing is encoded in the universal R-matrix for $DY(sl_2)$ which has the following form [KT]:

$$\mathcal{R} = R_+R_0R_-\ , \tag{15}$$

where

$$R_+ = \prod_{k\geq 0}^{\rightarrow} \exp(-\hbar e_k \otimes f_{-k-1}) \ , \qquad R_- = \prod_{k\geq 0}^{\leftarrow} \exp(-\hbar f_k \otimes e_{-k-1}) \ , \tag{16}$$

$$R_0 = \prod_{n\geq 0} \exp \hbar \mathrm{Res}_{u=v} (\frac{\mathrm{d}}{\mathrm{d}u} k^+(u)) \otimes k^-(v+2n\hbar+\hbar), \tag{17}$$

Here $k^\pm(u) = \frac{1}{\hbar} \ln h^\pm(u)$.

3 Central extension of $DY(sl_2)$

Let $d = \frac{\mathrm{d}}{\mathrm{d}u}$ be the operator of derivation of a spectral parameter: $d\,g(u) = \frac{\mathrm{d}}{\mathrm{d}u} g(u)$ and $T_x = \exp(xd)$ be the shift operator: $T_x g(u) = g(u+x)$.

Let us define semidirect products $Y^\pm \cdot \mathbf{C}[[d]]$, $\deg d = -1$, in a natural way:

(18)

$$[d, e^\pm(u)] = \frac{\mathrm{d}}{\mathrm{d}u} e^\pm(u), \qquad [d, h^\pm(u)] = \frac{\mathrm{d}}{\mathrm{d}u} h^\pm(u), \qquad [d, f^\pm(u)] = \frac{\mathrm{d}}{\mathrm{d}u} f^\pm(u).$$

Proposition 3.1 — *Semidirect products $Y^\pm \cdot \mathbf{C}[[d]]$ are Hopf algebras if we put*

$$\Delta(d) = d \otimes 1 + 1 \otimes d$$

The proof follows by induction from the observation that

$$\Delta(a_i) = a_i \otimes 1 + 1 \otimes a_i + \ \text{ terms of degree lower then } i$$

for $a = e, h, f$. Another argument is that the coproduct of $a^\pm(u)$ can be expressed again in terms of $a^\pm(u)$.

Denote by $\widehat{Y}^-$ the Hopf algebra $Y^- \cdot \mathbf{C}[[d]]$. Let $\widehat{Y}^+$ be the following Hopf algebra: $\widehat{Y}^+$ is a tensor product of Y^+ and of polynomial ring of central element c, $\deg c = 0$, as an algebra:

$$\widehat{Y}^+ = Y^+ \otimes \mathbf{C}[c],$$
$$\Delta_{\widehat{Y}^+}(c) = c \otimes 1 + 1 \otimes c$$

and

$$\Delta_{\widehat{Y}^+} a^+(u) = \left(\mathrm{Id} \otimes T^{-1}_{\hbar c \otimes 1}\right) \Delta_{Y^+} a^+(u) \tag{19}$$

for $a = e, h, f$. For instance,

$$\Delta_{\widehat{Y}^+} e^+(u) = e^+(u) \otimes 1 + \sum_{k=0}^{\infty} (-1)^k \hbar^{2k} \left(f^+(u+\hbar)\right)^k h^+(u) \otimes \left(e^+(u-\hbar c_1)\right)^{k+1},$$

where $c_1 = c \otimes 1$.

Proposition 3.2 — *There exists unique extension of the Hopf pairing from $Y^+ \otimes Y^-$ to $\widehat{Y}^+ \otimes \widehat{Y}^-$ satisfying the conditions:*

(i) $< c, d >= \frac{1}{\hbar}$;

(ii) *The Hopf pairing preserves the decompositions*

$$\widehat{Y}^+ = Y^+ \mathbf{C}[c], \qquad \widehat{Y}^- = Y^- \mathbf{C}[[d]]$$

Proof. Let $a^+(u) \in Y^+$, $b^-(u) \in Y^-$, $< a^+(u), b^-(v) >= f(u-v)$ and γ is a number, $\gamma \in \mathbf{C}$. Then, due to (i), (ii) and (19),

$$< a^+(u), e^{\gamma d} b^-(v) e^{-\gamma d} >=< a^+(u), e^{\gamma d} b^-(v) >=$$
$$< \Delta_{\widehat{Y}^+} a^+(u), e^{\gamma d} \otimes b^-(v) >= f(u-x-\gamma) =< a^+(u), b^-(v+\gamma) > .$$

which proves the compatibility of the extended pairing with the relations (18).

Definition 3.3 — *Central extension $\widehat{DY(sl_2)}$ of $DY(sl_2)$ is quantum double $D(\widehat{Y}^+)$ of the Hopf algebra $\widehat{Y}^+ = \widehat{Y}^+(sl_2)$.*

Equivalently, $\widehat{DY(sl_2)}$ is the double $D(\widehat{Y}^-)$ with opposite comultiplication.

The following theorem describes $\widehat{DY(sl_2)}$ explicitely as a Hopf algebra.

Theorem 3.4 — *$\widehat{DY(sl_2)}$ is isomorphic to a formal completion (see (10)) of the algebra with generators e_k, f_k, h_k, $k \in \mathbf{Z}$, d and central element c with the relations written in terms of generating functions:*

$$[d, e(u)] = \frac{d}{du} e(u), \qquad [d, f(u)] = \frac{d}{du} f(u), \qquad [d, h^\pm(u)] = \frac{d}{du} h^\pm(u),$$

$$\begin{aligned}
e(u)e(v) &= \frac{u-v+\hbar}{u-v-\hbar}\, e(v)e(u) \\
f(u)f(v) &= \frac{u-v-\hbar}{u-v+\hbar}\, f(v)f(u) \\
h^\pm(u)e(v) &= \frac{u-v+\hbar}{u-v-\hbar}\, e(v)h^\pm(u) \\
h^+(u)f(v) &= \frac{u-v-\hbar-\hbar c}{u-v+\hbar-\hbar c}\, f(v)h^+(u) \\
h^-(u)f(v) &= \frac{u-v-\hbar}{u-v+\hbar}\, f(v)h^-(u) \\
h^+(u)h^-(v) &= \frac{u-v+\hbar}{u-v-\hbar} \cdot \frac{u-v-\hbar-\hbar c}{u-v+\hbar-\hbar c}\, h^-(v)h^+(u) \\
[e(u), f(v)] &= \frac{1}{\hbar}\left(\delta(u-(v+\hbar c))h^+(u) - \delta(u-v)h^-(v)\right)
\end{aligned} \tag{20}$$

The comultiplication is given by the relations

$$\Delta(e^\varepsilon(u)) = e^\varepsilon(u)\otimes 1 + \sum_{k=0}^{\infty}(-1)^k\hbar^{2k}\big(f^\varepsilon(u+\hbar-\delta_{\varepsilon,+}\hbar c_1)\big)^k h^\varepsilon(u)\otimes$$
$$\otimes\big(e^\varepsilon(u-\delta_{\varepsilon,+}\hbar c_1)\big)^{k+1},$$
$$\Delta(h^\varepsilon(u)) = \sum_{k=0}^{\infty}(-1)^k(k+1)\hbar^{2k}\big(f^\varepsilon(u+\hbar-\delta_{\varepsilon,+}\hbar c_1)\big)^k h^\varepsilon(u)\otimes$$
$$\otimes h^\varepsilon(u-\delta_{\varepsilon,+}\hbar c_1)\big(e^\varepsilon(u+\hbar-\delta_{\varepsilon,+}\hbar c_1)\big)^k,$$

(21)

$$\Delta(f^\varepsilon(u)) = 1\otimes f^\varepsilon(u) + \sum_{k=0}^{\infty}(-1)^k\hbar^{2k}\big(f^\varepsilon(u+\delta_{\varepsilon,+}\hbar c_2)\big)^{k+1}\otimes h^\varepsilon(u)\big(e^\varepsilon(u+\hbar)\big)^k.$$

where $\varepsilon=\pm,\ \delta_{+,+}=1$ *and* $\delta_{-,+}=0$.

The proof of Theorem 3.4 reduces to explicit calculation of commutation relations in quantum double of $\widehat{Y}^+$. In abstract Sweedler notation for a double of a Hopf algebra A these relations have the following form:

$$a\cdot b = < a^{(1)}, b^{(1)} > < S^{-1}(a^{(3)}), b^{(3)} > \ b^{(2)}\cdot a^{(2)} \tag{22}$$

where $a\in A,\ b\in A^0,\ \Delta^2(x)=(\Delta\otimes Id)\Delta(x)=x^{(1)}\otimes x^{(2)}\otimes x^{(3)}$, S is antipode in A.

For the calculation of (22) we need the following partial information about Δ^2 and S^{-1} in $\widehat{Y}^\pm$ which one can deduce from (12) or directly by induction:

$$S^{-1}e^+(u) = -e(u-\hbar c)h^{-1}(u-\hbar c)\mod E^+\widehat{Y}^+F^+,$$
$$S^{-1}f^+(u) = -h^{-1}(u-\hbar c)f(u-\hbar c)\mod E^+\widehat{Y}^+F^+,$$
$$S^{-1}h^+(u) = h^{-1}(u-\hbar c)\mod E^+\widehat{Y}^+F^+ \tag{23}$$

and

$$\Delta^2 e^\pm(u) = K^\pm\big(e_1^\pm(u)+h_1^\pm(u)e_2^\pm(u)+h_1^\pm(u)h_2^\pm(u)e_3^\pm(u)-$$
$$-\hbar^2h_1^\pm(u)f_1^\pm(u-\hbar)(e_2^\pm(u))^2-2\hbar^2h_1^\pm(u)f_1^\pm(u-\hbar)e_2^\pm(u-\hbar)h_2^\pm(u)e_3^\pm(u)\big)\mod X^\pm$$
$$\Delta^2 f^\pm(u) = K^\pm\big(f_3^\pm(u)+f_2^\pm(u)h_3^\pm(u)+f_1^\pm(u)h_2^\pm(u)h_3^\pm(u)-$$
$$-\hbar^2(f_2^\pm(u))^2e_3^\pm(u-\hbar)h_3^\pm(u)-2\hbar^2f_1^\pm(u)h_2^\pm(u)f_2^\pm(u-\hbar)e_3^\pm(u-\hbar)h_3^\pm(u)\big)\mod X^\pm$$
$$\Delta^2h^\pm(u) = K^\pm\big(h_1^\pm(u)h_2^\pm(u)h_3^\pm(u)-2\hbar^2h_1^\pm(u)h_2^\pm(u)f_2^\pm(u-\hbar)e_3^\pm(u-\hbar)h_3^\pm(u)-$$
$$-2\hbar^2h_1^\pm(u)f_1^\pm(u-\hbar)e_2^\pm(u-\hbar)h_2^\pm(u)h_3^\pm(u)\big)\mod X^\pm \tag{24}$$

where

$$K^{+}=Id\otimes T^{-1}_{\hbar c_{1}}\otimes T^{-1}_{\hbar(c_{1}+c_{2})},\qquad K^{-}=Id\otimes Id\otimes Id,$$
$$X^{\pm}=(H^{\pm}F^{\pm}+H^{\pm})C^{\pm}\otimes\widehat{Y}^{\pm}\otimes E^{\pm 2}C^{\pm}+F^{\pm 2}C^{\pm}\otimes\widehat{Y}^{\pm}\otimes(E^{\pm}H^{\pm}+H^{\pm})C^{\pm},$$

$C^{+}=\mathbf{C}[c], C^{-}=\mathbf{C}[[d]]$, and, as usually, $a_1(u)$ means $a(u)\otimes 1\otimes 1$, $a_2(u)=1\otimes a(u)\otimes 1$, $a_3(u)=1\otimes 1\otimes a(u)$.

The substitution of (24) into (23) gives the following relations between generators of $\widehat{Y}^{+}$ and $\widehat{Y}^{-}$:

$$\begin{aligned}
[e^{+}(u),e^{-}(v)]&=-\hbar\frac{(e^{+}(u)-e^{-}(v))^{2}}{u-v}\ ,\\
[e^{+}(u),f^{-}(v)]&=-\frac{1}{\hbar}\frac{h^{+}(u)-h^{-}(v)}{u-v}\ ,\\
[e^{+}(v),h^{+}(v)]&=\hbar\frac{\{h^{-}(v),(e^{+}(u)-e^{-}(v))\}}{u-v}\ ,\\
[h^{+}(u),e^{-}(v)]&=-\hbar\frac{\{h^{+}(u),(e^{+}(u)-e^{-}(v))\}}{u-v}\ ,\\
[h^{+}(u),f^{-}(v)]&=\hbar\frac{\{h^{+}(u),(f^{+}(u)-f^{-}(v))\}}{u-v}\ ,\\
h^{+}(u)h^{-}(v)&=\frac{u-v+\hbar}{u-v-\hbar}\cdot\frac{u-v-\hbar-\hbar c}{u-v+\hbar-\hbar c}\ h^{-}(v)h^{+}(u)\\
[f^{-}(u),e^{+}(v)]&=-\frac{1}{\hbar}\left(\frac{h^{+}(u)}{u-v}-\frac{h^{-}(v)}{u-v-\hbar c}\right),\\
[f^{+}(u),h^{-}(v)]&=\hbar\frac{\{h^{-}(v),(f^{-}(v)-f^{+}(u))\}}{u-v-\hbar c}\ ,\\
[f^{+}(u),f^{-}(v)]&=\hbar\frac{(f^{+}(u)-f^{-}(v))^{2}}{u-v}\\
e^{+}(u)e^{\gamma d}&=e^{\gamma d}e^{+}(u-\gamma),\\
h^{+}(u)e^{\gamma d}&=e^{\gamma d}h^{+}(u-\gamma),\\
f^{+}(u)e^{\gamma d}&=e^{\gamma d}f^{+}(u-\gamma). \qquad (25)
\end{aligned}$$

After a change of variables

$$\begin{aligned}
f^{+}_{\text{new}}(u)&=f^{+}_{\text{old}}(u+\hbar c),\\
f(u)&=f^{+}_{\text{new}}(u)-f^{-}(u),\\
e(u)&=e^{+}(u)-e^{-}(u)
\end{aligned}$$

we get the relations (20) and comultiplication rules (21). The theorem is proved.

As a consequence of Proposition 3.2 and of the description of the universal R-matrix for $DY(sl_2)$ we have the explicit formula for the universal R-matrix for $\widehat{DY(sl_2)}$.

***Theorem 3.5* —**

$$\mathcal{R} = R_+ R_0 R_- \exp(\hbar c \otimes d) \ , \tag{26}$$

where

$$R_+ = \prod_{k\geq 0}^{\rightarrow} \exp(-\hbar e_k \otimes f_{-k-1}) \ , \qquad R_- = \prod_{k\geq 0}^{\leftarrow} \exp(-\hbar g_k \otimes e_{-k-1}) \ ,$$

$$g_k = \sum_{m=0}^{k} \frac{k!}{m!(k-m)!} f_{k-m} (\hbar c)^m, \tag{27}$$

$$R_0 = \prod_{n\geq 0} \exp \hbar \mathrm{Res}_{u=v} (\frac{\mathrm{d}}{\mathrm{d}u} k^+(u)) \otimes k^-(v + 2n\hbar + \hbar), \tag{28}$$

Here $k^{\pm}(u) = \frac{1}{\hbar} \ln h^{\pm}(u)$.

The universal R-matrix for $\widehat{DY(sl_2)}$ can be rewritten also in slightly more symmetric form:

$$\mathcal{R} = R'_+ R'_0 R'_-$$

where

$$R'_+ = R_+, \qquad R'_0 = R_0 \exp(\hbar c \otimes d), \qquad R'_- = \prod_{k\geq 0}^{\leftarrow} \exp(-\hbar f_k \otimes e_{-k-1}) \ .$$

4 L-operator presentation of $\widehat{DY(sl_2)}$

Let $\rho(x) = T_{-x}\rho(0)$ be the action of $\widehat{DY(sl_2)}$ in two-dimensional evaluation representation W_x of $DY(sl_2)$. In a basis $w_{\pm}$ this action looks as follows:

(29)

$$e_k w_+ = f_k w_- = 0, \ e_k w_- = x^k w_+, \ f_k w_+ = x^k w_-, \ h_k w_+ = x^k w_+, \ h_k w_- = -x^k w_-$$

Let

$$\begin{aligned} L^+(x) &= (\rho(x) \otimes Id) \exp(\hbar d \otimes c)(\mathcal{R}^{21})^{-1}, \\ L^-(x) &= (\rho(x) \otimes Id) \mathcal{R} \exp(-\hbar c \otimes d), \end{aligned} \tag{30}$$

and

$$R^{+}(x-y)=(\rho(x)\otimes\rho(y))\exp(\hbar d\otimes c)(\mathcal{R}^{21})^{-1},$$
$$R^{-}(x-y)=(\rho(x)\otimes\rho(y))\mathcal{R}\exp(-\hbar c\otimes d).$$

We have [KT]

$$R^{\pm}(u)=\rho^{\pm}(u)\cdot\begin{pmatrix}1&0&0&0\\0&\frac{u}{u+\hbar}&\frac{\hbar}{u+\hbar}&0\\0&\frac{\hbar}{u+\hbar}&\frac{u}{u+\hbar}&0\\0&0&0&1\end{pmatrix}\tag{31}$$

where

$$\rho^{\pm}(u)=\left(\frac{\Gamma(\mp\frac{u}{2\hbar})\Gamma(1\mp\frac{u}{2\hbar})}{\Gamma^{2}(\frac{1}{2}\mp\frac{u}{2\hbar})}\right)^{\pm1}$$

The Yang-Baxter equation on $\mathcal{R}$ implies the following relations on L^{+} and L^{-} (see [FR] for detailes in $U_q(\widehat{\mathbf{g}})$ case):

$$R^{\pm}_{12}(x-y)L^{\pm}_{1}(x)L^{\pm}_{2}(y)=L^{\pm}_{2}(y)L^{\pm}_{1}(x)R^{\pm}_{12}(x-y)\tag{32}$$
$$R^{+}_{12}(x-y-\hbar c)L^{-}_{1}(x)L^{+}_{2}(y)=L^{+}_{2}(y)L^{-}_{1}(x)R^{+}_{12}(x-y)\tag{33}$$

where $L_1=L\otimes Id$, $L_2=Id\otimes L$. The properties of comultiplication for $\mathcal{R}$:

$$(\Delta\otimes Id)\mathcal{R}=\mathcal{R}^{13}\mathcal{R}^{23},\qquad (Id\otimes\Delta)\mathcal{R}=\mathcal{R}^{13}\mathcal{R}^{12}$$

imply the comultiplication rules for $L^{\pm}$:

$$\Delta' L^{+}(x)=L^{+}(x-\hbar c_2)\otimes L^{+}(x),\qquad \text{or}\quad \Delta l^{+}_{ij}(u)=\sum_{k}l^{+}_{kj}(u)\otimes l^{+}_{ik}(u-\hbar c_1),$$

$$\Delta' L^{-}(x)=L^{-}(x)\otimes L^{-}(x),\qquad \text{or}\quad \Delta l^{-}_{ij}(u)=\sum_{k}l^{-}_{kj}(u)\otimes l^{-}_{ik}(u).$$

The explicit formula for the universal R-matrix and (29) express Gauss factors of the L-operators in terms of Drinfeld generators:

$$L^{+}(x)=\begin{pmatrix}1&\hbar f^{+}(x-\hbar c)\\0&1\end{pmatrix}\begin{pmatrix}k^{+}_{1}(x)&0\\0&k^{+}_{2}(x)\end{pmatrix}\begin{pmatrix}1&0\\\hbar e^{+}(x)&1\end{pmatrix},$$

$$L^{-}(x)=\begin{pmatrix}1&\hbar f^{-}(x)\\0&1\end{pmatrix}\begin{pmatrix}k^{-}_{1}(x)&0\\0&k^{-}_{2}(x)\end{pmatrix}\begin{pmatrix}1&0\\\hbar e^{-}(x)&1\end{pmatrix},$$

with $h^{\pm}(x) = k_2^{\pm}(x)^{-1}k_1(x)$, $k_1^{\pm}(x)k_2^{\pm}(x-\hbar) = 1$ analogous to Ding-Frenkel formulas for $U_q(\widehat{gl}_n)$ [DF].

Note that Frenkel and Reshetikhin use more symmetric form of the equation (33), using the shifts of spectral parameter in both sides of equation. One can get analogous form via twisting of comultiplication in $\widehat{DY(sl_2)}$ by elements $\exp(\alpha\hbar c\otimes d)$ and $\exp(\beta\hbar d\otimes c)$, $\alpha,\beta\in\mathbf{C}$. For quantum affine algebras such renormalizations are initiated by the condition of consistency with Cartan involution in quantum affine algebra. There is no analogous motivation in our case so we do not use these twists.

5 Basic representations of $\widehat{DY(sl_2)}$

Let $\mathscr{H}$ be Heisenberg algebra generated by free bosons with zero mode $a_{\pm n}, n\geq 1, a_0 = \alpha$ and $p = \partial_\alpha$ with commutation relations

$$[a_n, a_m] = n\delta_{n+m,0}, \qquad [p, a_0] = 2. \tag{34}$$

In the following we use the generating functions

$$a_+(z) = \sum_{n\geq 1}\frac{a_n}{n}z^{-n} - p\log z, \qquad a_-(z) = \sum_{n\geq 1}\frac{a_{-n}}{n}z^{n} + \frac{a_0}{2}, \tag{35}$$

$$a(z) = a_+(z) - a_-(z), \qquad \phi_\pm(z) = \exp a_\pm(z). \tag{36}$$

They satisfy the relation

$$[a_+(z), a_-(u)] = -\log(z-w)$$

Let $\Lambda_i, i = 0,1$ be formal power extensions of the Fock spaces:

$$\Lambda_i = \mathbf{C}[[a_{-1},\dots,a_{-n},\dots]]\otimes\left(\oplus_{n\in\mathbf{Z}+\frac{i}{2}}\mathbf{C}e^{n\alpha}\right).$$

The following relations define an action of $\widehat{DY(sl_2)}$ on Λ_i with central charge $c = 1$. We call them basic representations of $\widehat{DY(sl_2)}$:

$$\begin{aligned}
e(u) &= \exp\left(\sum_{n=1}^{\infty}\frac{a_{-n}}{n}\left[(u-\hbar)^n + u^n\right]\right)\exp\left(-\sum_{n=1}^{\infty}\frac{a_n}{n}u^{-n}\right)e^{\alpha}u^{\partial_\alpha}\\
f(u) &= \exp\left(-\sum_{n=1}^{\infty}\frac{a_{-n}}{n}\left[(u+\hbar)^n + u^n\right]\right)\exp\left(\sum_{n=1}^{\infty}\frac{a_n}{n}u^{-n}\right)e^{-\alpha}u^{-\partial_\alpha}\\
h^-(u) &= \exp\left(\sum_{n=1}^{\infty}\frac{a_{-n}}{n}\left[(u-\hbar)^n - (u+\hbar)^n\right]\right)\\
h^+(u) &= \exp\left(\sum_{n=1}^{\infty}\frac{a_n}{n}\left[(u-\hbar)^{-n} - u^{-n}\right]\right)\left(\frac{u}{u-\hbar}\right)^{\partial_\alpha}
\end{aligned} \tag{37}$$

The action of $e^{\gamma d}$, $\gamma \in \mathbf{C}$ is defined by the prescriptions

$$e^{\gamma d} \cdot (1 \otimes 1) = 1 \otimes 1 \tag{38}$$

and

$$\begin{aligned}
e^{\gamma d} a_{-n} e^{-\gamma d} &= \sum_{k \geq 0} \frac{(n+k-1)!}{(n-1)!k!} a_{-(n+k)} \gamma^k, \qquad n \geq 1,\\
e^{\gamma d} a_n e^{-\gamma d} &= \sum_{0 \leq k < n} (-1)^k \frac{(n)!}{(n-k)!k!} a_{n-k} \gamma^k + (-1)^n \gamma^n p, \qquad n \geq 1,\\
e^{\gamma d} a_0 e^{-\gamma d} &= a_0 + 2 \left(\sum_{n \geq 1} \frac{a_{-n}}{n} \gamma^n \right), \qquad e^{\gamma d} p e^{-\gamma d} = p
\end{aligned} \tag{39}$$

Note that the relations (39) define an automorphism of Heisenberg algebra $T_\gamma a(z) = a(z+\gamma)$.

In terms of generating functions $\phi_\pm(z)$ the action of $\widehat{DY(sl_2)}$ in basic representations has the following compact form:

$$\begin{aligned}
e(u) &= \phi_-(u-\hbar)\phi_-(u)\phi_+^{-1}(u),\\
f(u) &= \phi_-^{-1}(u+\hbar)\phi_-^{-1}(u)\phi_+(u),\\
h^+(u) &= \phi_+(u-\hbar)\phi_+^{-1}(u),\\
h^-(u) &= \phi_-(u-\hbar)\phi_-^{-1}(u+\hbar),\\
e^{\gamma d}\phi_\pm(u) &= \phi_\pm(u+\gamma)e^{\gamma d}
\end{aligned} \tag{40}$$

The relations (37), (29), (21) give possibility to write down bozonized expressions for intertwining operators of type one and two $\Phi(z) : \Lambda_i \to \Lambda_{1-i} \otimes W_z$, $\Psi(z) : \Lambda_i \to W_z \otimes \Lambda_{1-i}$ analogous to the case of $U_q(\widehat{sl}_2)$ [DFJMN]. More concretely, let

$$\Phi(z) : \Lambda_i \to \Lambda_{1-i} \otimes W_z, \quad \Psi(z) : \Lambda_i \to W_z \otimes \Lambda_{1-i}, \quad x \in \widehat{DY(sl_2)}$$

satisfy the relations

$$\Phi(z)x = \Delta(x)\Phi(z), \quad \Psi(z)x = \Delta(x)\Psi(z).$$

The components of intertwining operators are defined as follows

$$\Phi(z)v = \Phi_+(z)v \otimes v_+ + \Phi_-(z)v \otimes v_-, \quad \Psi(z)v = v_+ \otimes \Psi_+(z)v + v_- \otimes \Psi_-(z)v,$$

where $v \in \Lambda_0$ or in Λ_1. Then, for instance, $\Psi_-(z)$ is the solution of the system of equations

$$\Phi_-(z)h^+(u) = \frac{u-z-2\hbar}{u-z-\hbar} h^+(u)\Phi_-(z),$$

$$\Phi_-(z)h^-(u) = \frac{u-z-\hbar}{u-z}h^-(u)\Phi_-(z), \tag{41}$$

$$\Phi_-(z)e(u) = e(u)\Phi_-(z),$$

and

$$\Phi_+(z) = \Phi_-(z)f_0 - f_0\Phi_-(z).$$

Formal solution of (41) is

$$\Phi_-(z) = \phi_-(z+\hbar)(-1)^{\frac{p}{2}}\prod_{k=0}^{\infty}\frac{\phi_+(z-\hbar-2k\hbar)}{\phi_+(z-2k\hbar)} \tag{42}$$

modulo normalization factors depending on i.
S. Pakuliak suggested the following regularization of the formal solution (42):

$$\begin{aligned}\Phi_-(z) = \exp\left(\sum_{n=1}^{\infty}\frac{a_{-n}}{n}(z+\hbar)^n\right)e^{\alpha/2}(2\hbar)^{\partial_\alpha/2}\left(\Gamma(\frac{1}{2}-\frac{z}{2\hbar})\Gamma(-\frac{z}{2\hbar})\right)^{\partial_\alpha}\\ \times\prod_{k=0}^{N}\exp\left(-\sum_{n=1}^{\infty}\frac{a_n}{n}\left[(z-2k\hbar)^{-n}-(z-\hbar-2k\hbar)^{-n}\right]\right)\end{aligned} \tag{43}$$

Let us note that our form of presentation (40) for basic representations and (42) for corresponding intertwining operators $\Phi(z)$ and $\Psi(z)$ looks to be quite general. In particular, one can see that the Frenkel-Jing [FJ] formulas for basic representations of $U_q(\widehat{sl}_2)$ and corresponding expressions for intertwining operators $\Phi(z)$ and $\Psi(z)$ [JM] could be rewritten in a form analogous to (40) and (42) using usual boson field $a(z)$ and multiplicative shifts of spectral parameter.

It will be interesting to equip basic representations of $\widehat{DY(sl_2)}$ with invariant bilinear form and with a structure of topological representation.

An application of intertwining operators for $\widehat{DY(sl_2)}$ to calculation of correlation functions in $su(2)$-invariant Thirring model is given in forthcoming paper [KLP].

6 The general case

The arguments of section 3 prove that there exists central extension $\widehat{DY(\mathbf{g})}$ of the Yangian double for any simple finite-dimensional Lie algebra $\mathbf{g}$. Technical computations of the relations in the double are more complicated in general case and we did not make them up to the end. Nevertheless in this section we communicate the algebraic structure of $\widehat{DY(\mathbf{g})}$.

Let $\mathbf{g}$ be a simple Lie algebra with a standard Cartan matrix $A = (a_{ij})_{i,j=0}^{r}$, a system of simple roots $\Pi := \{\alpha_1, \dots, \alpha_2\}$ and a system of positive roots $\Delta_+(\mathbf{g})$. Let

$e_i := e_{\alpha_i}$, $h_i := h_{\alpha_i}$, $f_i := f_{\alpha_i} := e_{-\alpha_i}$, $(i = 1, \ldots, r)$, be Chevalley generators and $\{e_\gamma, f_\gamma\}$, $(\gamma \in \Delta)$, be Cartan-Weyl basis in $\mathbf{g}$, normalized so that $(e_\alpha, f_\alpha) = 1$.

Let us first describe $\widehat{DY(\mathbf{g})}$ as an algebra.

Central extension $\widehat{DY(\mathbf{g})}$ of the Yangian $Y(\mathbf{g})$ is a formal completion (see (10)) of the algebra generated by the elements $e_{ik} := e_{\alpha_i,k}$, $h_{ik} := h_{\alpha_i,k}$, $f_{ik} := f_{\alpha_i,k}$, $(i = 1, \ldots, r;\ k = 0, 1, 2, \ldots)$, c and d, $\deg e_{ik} = \deg f_{ik} = \deg h_{ik} = k$, $\deg c = 0, \deg d = -1$. The relations are written in terms of generating functions

$$e_i^{\pm}(u) := \pm \sum_{\substack{k \ge 0 \\ k<0}} e_{ik} u^{-k-1}, f_i^{\pm}(u) := \pm \sum_{\substack{k \ge 0 \\ k<0}} f_{ik} u^{-k-1}, h_i^{\pm}(u) := 1 \pm \hbar \sum_{\substack{k \ge 0 \\ k<0}} h_{ik} u^{-k-1},$$

$$e_i(u) = e_i^+(u) - e_i^-(u), \qquad f_i(u) = f_i^+(u) - f_i^-(u):$$

$$[d, e_i(u)] = \frac{\mathrm{d}}{\mathrm{d}u} e_i(u), \qquad [d, f_i(u)] = \frac{\mathrm{d}}{\mathrm{d}u} f_i(u), \qquad [d, h_i^{\pm}(u)] = \frac{\mathrm{d}}{\mathrm{d}u} h_i^{\pm}(u),$$

$$\begin{aligned}
e_i(u)e_j(v) &= \frac{u-v+\hbar b_{ij}}{u-v-\hbar b_{ij}}\ e_i(v)e_j(u)\\
f_i(u)f_j(v) &= \frac{u-v-\hbar b_{ij}}{u-v+\hbar b_{ij}}\ f_i(v)f_j(u)\\
h_i^{\pm}(u)e_j(v) &= \frac{u-v+\hbar b_{ij}}{u-v-\hbar b_{ij}}\ e_j(v)h_i^{\pm}(u)\\
h_i^{+}(u)f_j(v) &= \frac{u-v-\hbar b_{ij}-\hbar b_{ij}c}{u-v+\hbar b_{ij}-\hbar b_{ij}c}\ f_j(v)h_i^{+}(u)\\
h_i^{-}(u)f_j(v) &= \frac{u-v-\hbar b_{ij}}{u-v+\hbar b_{ij}}\ f_j(v)h_i^{-}(u)\\
h_i^{+}(u)h_j^{-}(v) &= \frac{u-v+\hbar b_{ij}}{u-v-\hbar b_{ij}} \cdot \frac{u-v-\hbar b_{ij}-\hbar b_{ij}c}{u-v+\hbar b_{ij}-\hbar b_{ij}c}\ h_j^{-}(v)h_i^{+}(u)\\
[e_i(u), f_j(v)] &= \frac{\delta_{i,j}}{\hbar}\left(\delta(u-(v+\hbar b_{ii}c))h_i^+(u) - \delta(u-v)h_i^-(v)\right),
\end{aligned}$$

$$(44)\qquad \begin{cases} \mathrm{Sym}_{\{k\}}[e_i(u_{k_1})[e_i(u_{k_2})\ldots[e_i(u_{k_{n_{ij}}}, e_j(v)]\ldots]] = 0 \\ \mathrm{Sym}_{\{k\}}[f_i(u_{k_1})[f_i(u_{k_2})\ldots[f_i(u_{k_{n_{ij}}}, f_j(v)]\ldots]] = 0 \end{cases} \qquad \text{for } i \neq j\ ,$$

Here $b_{ij} = \frac{1}{2}(\alpha_i, \alpha_j)$, $n_{ij} = 1 - A_{ij}$.

Hopf algebra $\widehat{DY(\mathbf{g})}$ is the double of a Hopf algebra $\widehat{Y}^+(\mathbf{g})$ which is isomorphic to tensor product $Y(\mathbf{g}) \otimes \mathbf{C}[c]$ as an algebra. The comultiplication in $\widehat{Y}^+(\mathbf{g})$ is given by the following rules, where we identify elements of $U(\mathbf{g})$ with corresponding elements of $Y(\mathbf{g})$ generated by e_{i0}, h_{i0}, f_{i0}.

$$\Delta(c) = c \otimes 1 + 1 \otimes c,$$

$$\Delta(x) = x\otimes 1 + 1\otimes x\ , \qquad x\in \mathbf{g}\ ,$$
$$\Delta(e_{i1}) = e_{i1}\otimes 1 + 1\otimes e_{i1} + \hbar(h_{i0}-c)\otimes e_{i0} - \hbar \sum_{\gamma\in\Delta_+(g)} f_\gamma \otimes [e_{\alpha_i}, e_\gamma]\ ,$$
$$\Delta(f_{i1}) = f_{i1}\otimes 1 + 1\otimes f_{i1} + \hbar f_{i0}\otimes (h_{i0}+c) + \hbar \sum_{\gamma\in\Delta_+(g)} [f_{\alpha_i}, f_\gamma]\otimes e_\gamma\ ,$$

$$\Delta(h_{i1}) = h_{i1}\otimes 1 + 1\otimes h_{i1} + \hbar(h_{i0}-c)\otimes h_{i0} - \hbar \sum_{\gamma\in\Delta_+(g)} (\alpha_i,\gamma) f_\gamma\otimes e_\gamma\ . \tag{45}$$

There is also partial information about comultiplication for basic fields in $\widehat{DY(\mathbf{g})}$:

$$\Delta(e_i^\pm(u)) = A^\pm\left(e_i^\pm(u)\otimes 1 + h_i^\pm(u)\otimes e_i^\pm(u)\right) \quad \bmod F^\pm\widehat{Y}^\pm(\mathbf{g})\otimes (E^\pm)^2,$$
$$\Delta(f_i^\pm(u)) = B^\pm\left(1\otimes f_i^\pm(u) + f_i^\pm(u)\otimes h_i^\pm(u)\right) \quad \bmod (F^\pm)^2\otimes E^\pm\widehat{Y}^\pm(\mathbf{g}),$$

$$\Delta(h_i^\pm(u)) = A^\pm h_i^\pm(u)\otimes h_i^\pm(u) \quad \bmod F^\pm\widehat{Y}^\pm(\mathbf{g})\otimes E^\pm\widehat{Y}^\pm(\mathbf{g}) \tag{46}$$

where

$$A^+ = Id\otimes T^{-1}_{\hbar c\otimes 1}, \qquad B^+ = T_{1\otimes\hbar c}\otimes Id, \qquad A^- = B^- = Id\otimes Id,$$

$E^\pm$ and $F^\pm$ are subalgebras generated by the components of $e_i^\pm(u)$ and $f_i^\pm(u)$, $i=1,2,\ldots,r=\mathrm{rank}\mathbf{g}$, $\widehat{Y}^-(\mathbf{g}) = (\widehat{Y}^+(\mathbf{g}))^0$.

Note that, to the contrary to sl_2 case, there is no explicit formula for the comultiplication in $\widehat{Y}^-(\mathbf{g})$ except that it is dual to the multiplication in $\widehat{Y}^\pm(\mathbf{g})$ with respect to their Hopf pairing. This pairing preserves the same decompositions as in sl_2 case and for the basic fields we have [KT]

$$< e_i^+(u), f_j^-(v) >= \frac{\delta_{i,j}}{\hbar(u-v)}, \qquad < f^+(u), e^-(v) >= \frac{\delta_{i,j}}{\hbar(u-v)},$$
$$< h^+(u), h^-(v) >= \frac{u-v+\hbar b_{ij}}{u-v-\hbar b_{ij}} \quad \text{and} \quad < c,d >= \frac{1}{\hbar}.$$

Analogously to sl_2 case, the universal R-matrix $\mathcal{R}$ for $\widehat{DY(\mathbf{g})}$ could be reconstructed from that of $DY(\mathbf{g})$ by the same procedure. Unfortunately, complete exact formula for $\mathcal{R}$ is not known in general case.

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References

[DFJMN] B. Davies, O. Foda, M. Jimbo, T. Miwa and A. Nakayashiki. *Diagonalization of the XXZ Hamiltonian by vertex operators.* Comm. Math. Phys. 151 (1993), 89–153.

[D1] V. G. Drinfeld. *Hopf algebras and quantum Yang-Baxter equation.* Soviet Math. Dokl. 283 (1985), 1060–1064.

[D2] ———. *A new realization of Yangians and quantized affine algebras.* Soviet Math. Dokl. 32 (1988), 212–216.

[DF] J. Ding and I. B. Frenkel. *Isomorphism of two realizations of quantum affine algebras $U_q(gl(n))$.* Commun. Math. Phys., 156 (1993), 277–300.

[FRT] L. D. Faddeev, N. Yu. Reshetikhin and L. A. Takhtajan. *Quantization of Lie groups and Lie algebras.* Algebra and Analysis 1 (1989), 178–201.

[FJ] I. B. Frenkel and N. Jing. *Vertex operators of quantum affine algebras.* Proc. Nat. Acad. Sci. USA, 85 (1988), 9373–9377.

[FR] I. B. Frenkel and N. Yu. Reshetikhin. *Quantum affine algebras and holonomic difference equations.* Commun. Math. Phys., 146 (1992), 1–60.

[JM] M. Jimbo and T. Miwa. *Algebraic Analysis of Solvable Lattice Models.* Conference Board of the Math. Sci., Regional Conference Series in Mathematics, 85 (1995).

[KLP] S. Khoroshkin, D. Lebedev and S. Pakuliak. *Intertwining Operators for the Central Extension of the Yangian Double.* Preprint DFTUZ/95-28, ITEP-TH-15/95.

[KT] S. Khoroshkin and V. Tolstoy. *Yangian Double* Lett. Math. Phys., to appear.

[M] A. I. Molev. *Private communication.*

[RS] N. Yu. Reshetikhin and M. A. Semenov-Tyan-Shansky. *Central extensions of quantum current groups*, Let. Math. Phys. 19 (1990), 133–142.

[S] F. A. Smirnov. *Dynamical symmetries of massive integrable models* Jour. Mod. Phys. A, 7 suppl. 1B (1992), 813–838.