ISSN 1472-2739 (on-line) 1472-2747 (printed)

Algebraic & Geometric Topology
Volume 4 (2004) 121–131
Published: 15 March 2004

Smith Theory for algebraic varieties

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Abstract We show how an approach to Smith Theory about group actions on CW–complexes using Bredon cohomology can be adapted to work for algebraic varieties.

AMS Classification 57S17; 14F20

Keywords Smith Theory, Bredon cohomology, coefficient system, variety

1 Introduction

Peter May described in [8] a version of Smith Theory based on Bredon cohomology, so it really applies to any complex of projective coefficient systems rather than just a topological space. Later Jeremy Rickard in [9] showed how to associate a complex of p-permutation modules to a group action on a variety in such a way that the cohomology of this complex is the étale cohomology of the variety. We show how to generalize this to obtain a complex of projective coefficient systems. Thus Smith Theory becomes available for algebraic varieties, even over fields of finite characteristic. Our framework is also sufficient to apply to varieties methods of Borel, Swan and others based on equivariant cohomology, although we do not set out the details here.

2 Coefficient systems

A coefficient system L on a group G over a ring R is a functor from the right orbit category of G to R-modules. In more concrete terms, it consists of a collection of R-modules $L(H)$, one for each subgroup $H \leq G$ together with R-linear restriction maps $\mathrm{res}^H_K\colon L(H) \to L(K)$ for each $K \leq H \leq G$ and conjugation maps $c_{g,H}\colon L(H) \to L({}^gH)$ for each $g \in G$ and $H \leq G$.

These must satisfy the identities:

(1) $\mathrm{res}^H_H = \mathrm{id}, \quad H \le G$;

(2) $\mathrm{res}^K_J \, \mathrm{res}^H_K = \mathrm{res}^H_J, \quad J \le K \le H \le G$;

(3) $c_{g_1, {}^{g_2}H} c_{g_2,H} = c_{g_1 g_2, H}, \quad H \le G, \quad g_1, g_2 \in G$;

(4) $\mathrm{res}^{{}^gH}_{{}^gK} \, c_{g,H} = c_{g,K} \, \mathrm{res}^H_K, \quad K \le H \le G, \quad g \in G$;

(5) $c_{h,H} = \mathrm{id}, \quad H \le G, \quad h \in H$.

In particular, the conjugation maps make $L(H)$ into a left $RN_G(H)/H$-module.

A morphism $f\colon L \to M$ is a collection of R-linear maps $f(H)\colon L(H) \to M(H)$ which commute with the res and c.

The coefficient systems on G over R form an abelian category, which we denote by $\mathrm{CS}_R(G)$. If $H \le G$ there is a forgetful map $\mathrm{Res}^G_H\colon \mathrm{CS}_R(G) \to \mathrm{CS}_R(H)$

Examples (1) The constant coefficient system $\bar{R}$, which is just R on each evaluation and all the maps are the identity.

(2) The fixed point coefficient system $V^?$, where V is a left RG-module and the notation indicates that the evaluation on $H \le G$ is the fixed point submodule V^H. Restriction is inclusion and conjugation is multiplication by $g \in G$. These have the important property that $\mathrm{Hom}_{\mathrm{CS}_R(G)}(L, V^?) \cong \mathrm{Hom}_{RG}(L(1), V)$.

(3) A variation on $V^?$ is V_0, which takes the value V on 1 and 0 elsewhere.

(4) The systems $R[X^?]$, where X is a left G-set and the evaluation at H is the free R-module on the fixed point set X^H.
The particular cases $R[G/H^?]$ have the important property

$$\mathrm{Hom}_{\mathrm{CS}_R(G)}(R[G/H^?], L) \cong L(H).$$

It follows that they are projective and that they provide enough projectives. Thus every projective is a summand of a sum of these.

For more information on coefficient systems see [10].

Given a set of coefficient systems I it is convenient to define $\mathrm{add}(I)$ to be the full subcategory of $\mathrm{CS}_R(G)$ in which the objects are isomorphic to a summand of a coefficient system of the form $L_1 \oplus \ldots \oplus L_n, \quad L_i \in I$.

Thus the subcategory $\mathrm{proj}(\mathrm{CS}_R(G))$ of finitely generated projective coefficient systems is the same as $\mathrm{add}(\{R[G/H^?] : H \le G\})$.

If X is a G–CW–complex then there is a complex of coefficient systems $C[X^?]$ associated to it, in which $C_n[X] = R[(X_n)^?]$ where X_n is the G-set of n-cells in X and the boundary morphisms are defined in the usual way.

The Bredon cohomology of X with coefficients in a coefficient system L, as defined in [3], is $H^*_G(X, L) = H^*(\mathrm{Hom}_{\mathrm{CS}_R(G)}(C[X^?], L))$.

Examples (1) $H^*_G(X, (RG)^?) \cong H^*(X, R)$, the usual CW–cohomology,

(2) $H^*_G(X, R_G) \cong H^*(X^G, R)$, where R_G takes the value R on G and 0 elsewhere.

(3) $H^*_G(X, \bar{R}) \cong H^*(X/G, R)$,

(4) More generally we can regard $H^*_?(X, R)$ as a coefficient system itself under the natural restriction and conjugation maps, and then we have $H^*_?(X, \bar{R}) \cong H^*(X/?, R)$.

The dual concept to that of a coefficient system we term an efficient system, in which the restriction maps go in the opposite direction. $E(H)$ is now a right $N_G(H)$-module, although we could remedy this by taking the contragredient instead of the dual. The category of efficient systems for G over R is denoted by $\mathrm{ES}_R(G)$.

If R is self-injective then applying $\mathrm{Hom}_R(-, R)$ provides a duality between the subcategories taking values in finitely generated modules.

The dual of $R[G/H^?]$ is denoted by $R[G/H^?]^*$. The evaluation on $K \leq G$ can be thought of as the functions on the fixed point set $(G/H)^K$ and the restriction maps just restrict the functions. If R is self injective then $R[G/H^?]^*$ is injective.

$\mathrm{CS}_R(G)$ can also be viewed as the category of modules over an R-algebra $C_R(G)$ of finite rank over R, (cf. [2]). Similarly $\mathrm{ES}_R(G)$ is equivalent to the category of modules over another R-algebra $E_R(G)$.

3 Varieties

From now on k is an algebraically closed field and in this section X is a separated scheme of finite type over k.

Let A be a torsion Artin algebra and let $\mathcal{F}$ be a constructible sheaf of A-modules over X. Let stalks($\mathcal{F}$) denote the set of stalks of $\mathcal{F}$ at the k-rational points. This contains only a finite number of isomorphism classes of A-modules.

Recall that $R\Gamma_c(X, \mathcal{F})$ is a complex of A-modules, natural as an object of the derived category $D(A - \mathrm{Mod})$, whose homology is étale cohomology with compact supports $H^*_c(X, \mathcal{F})$.

Our main tool will be the following result from [9]:

Theorem 3.1 (Rickard) *There is a complex of modules in* $\operatorname{add}(\operatorname{stalks}(\mathcal{F}))$ *of finite type, which we denote by* $\Omega_c(X,\mathcal{F})$. *It is well defined up to homotopy equivalence. It has the following properties:*

(1) $\Omega_c(X,\mathcal{F})$ *is isomorphic to* $R\Gamma_c(X,\mathcal{F})$ *in* $D(A-\mathrm{Mod})$;

(2) $\mathcal{F}\mapsto\Omega_c(X,\mathcal{F})$ *is a functor from constructible sheaves of* A*-modules over* X *to* $K^b(A-\mathrm{mod})$;

(3) *If* $f\colon Y\to X$ *is a finite morphism of separated schemes of finite type over* k *then there is an induced map* $\Omega_c(X,\mathcal{F})\to\Omega_c(Y,f^*\mathcal{F})$;

(4) *If* B *is also a torsion Artin algebra and* L *is a functor* $\operatorname{add}(\operatorname{stalks}(\mathcal{F}))\to B-\mathrm{mod}$ *then* $L\Omega_c(X,\mathcal{F})\cong\Omega_c(X,\tilde{L}\mathcal{F})$, *where* $\tilde{L}\mathcal{F}$ *denotes the sheafification of the presheaf* $L\mathcal{F}$.

We will apply this in the case that $R=\mathbb{Z}/\ell^n$ and $A=E_R(G)$.

We suppose that a finite group G acts on X with quotient variety $Y=X/G$ and projection map $\rho\colon X\to Y$. We let $\mathcal{F}=\mathcal{F}_X$ be the sheafification of the presheaf that sends a Zariski open set $U\subseteq Y$ in the Zariski topology to $R[(\pi_0(\rho^{-1}U))^?]^*$, where $\pi_0(\rho^{-1}U)$ is the G-set of components of $\rho^{-1}U$. (This extends to the étale site on X by evaluating on the image of an étale map $U\to X$.) Then $\operatorname{stalks}(\mathcal{F})$ consists of injective modules.

Theorem 3.1 produces a complex of injective efficient systems of finite type $\Omega_c(Y,\mathcal{F})$. These complexes for different n can be pieced together in such a way that we can take the inverse limit and obtain a complex of finite type of efficient systems in $\operatorname{add}(\{\hat{\mathbb{Z}}_\ell[G/H^?]^* : H \text{ the stabilizer of a } k\text{-rational point}\})$ as in [9]. The dual of this by $\operatorname{Hom}_{\hat{\mathbb{Z}}_\ell}(-,\hat{\mathbb{Z}}_\ell)$ is the complex that we will denote by $C[X^?]$.

Theorem 3.2 *For any* $H\leq G$, $C[X^?](H)^*\cong R\Gamma_c(X^H,\hat{\mathbb{Z}}_\ell)$.

In other words $C[X^?](H)$ is a complex whose dual has cohomology $H^*_c(X^H,\hat{\mathbb{Z}}_\ell)$. We can therefore think of it as the analogue of the complex $C[X^?]$ for the Bredon cohomology of a G–CW–complex.

Since $C[X^H](1)\cong C[X^?](H)$ our notation is justified and, after the proof is complete, we will write $C[X^H]$ instead of $C[X^?](H)$.

We will prove theorem 3.2 as a corollary of some more general results.

Notice that $C[X^?]$ is natural with respect to group homomorphisms $f\colon H\to G$ for which the kernel acts trivially on X.

Let $\mathcal{A}$ be a set of subgroups of G closed under supergroups and conjugation. Let $S_{\mathcal{A}}X = \bigcup_{J\in\mathcal{A}} X^J$.

Define $L_{\mathcal{A}}\colon \mathrm{CS}_R(G) \to \mathrm{CS}_R(G)$ by taking $L_{\mathcal{A}}C$ to be the smallest subsystem of C that is equal to $C(H)$ for all $H \in \mathcal{A}$. Then

$$\begin{aligned} L_{\mathcal{A}}R[G/H^?] &= \begin{cases} R[G/H^?] & H\in\mathcal{A} \\ 0 & \text{otherwise} \end{cases} \\ &= R[(S_{\mathcal{A}}G/H)^?]. \end{aligned}$$

So $L_{\mathcal{A}}$ induces a functor $L_{\mathcal{A}}\colon \ \mathrm{add}(\{R[G/H^?]; H\le G\}) \to \mathrm{add}(\{R[G/H^?]; H \in \mathcal{A}\})$.

Proposition 3.3 *$L_{\mathcal{A}}C[X^?]$ is homotopy equivalent to $C[(S_{\mathcal{A}}X)^?]$.*

Proof It is sufficient to prove the analogous statement for $R = \mathbb{Z}/\ell^n$.

By 3.1, $L_{\mathcal{A}}C[X^?]^* \cong \Omega_c(Y, \tilde{L}_{\mathcal{A}}\mathcal{F}_1)$, where $\mathcal{F}_1$ is the sheafification of $\mathcal{F}_1'\colon U \mapsto \Gamma((\pi_0(\rho^{-1}U))^?, R)$. So $\tilde{L}_{\mathcal{A}}\mathcal{F}_1$ is the sheafification of $U \mapsto \Gamma((S_{\mathcal{A}}\pi_0(\rho^{-1}U))^?, R)$.

Now $C[(S_{\mathcal{A}}X)^?]^* \cong \Omega_c((S_{\mathcal{A}}X)/G, \mathcal{F}_{S_{\mathcal{A}}X}) \cong \Omega_c(Y, \mathcal{F}_2)$, where $\mathcal{F}_2$ is the sheafification of $\mathcal{F}_2'\colon U \mapsto \Gamma((\pi_0(\rho^{-1}U \cap S_{\mathcal{A}}X))^?, R)$.

Inclusion of fixed points gives a map $\tilde{L}_{\mathcal{A}}\mathcal{F}_1' \to \mathcal{F}_2'$, which induces an isomorphism on the stalks and hence an isomorphism of sheaves. $\square$

Lemma 3.4 $C[X^?](1)^* \cong R\Gamma_c(X, R)$

Proof Again it is enough to work over $\mathbb{Z}/\ell^n$.

Considering the functor "evaluate at 1", we find $\Omega_c(Y,\mathcal{F})(1) \cong \Omega_c(Y, \widetilde{\mathcal{F}(1)})$, where $\widetilde{\mathcal{F}(1)}$ is the sheafification of $U \mapsto \Gamma(\pi_0(\rho^{-1}U), R)$, which is just ρ_*R.

Finally $\Omega_c(Y, \rho_*R) \cong R\Gamma_c(Y, \rho_*R) \cong R\Gamma_c(X, R)$. $\square$

Remark The above lemma shows that $C[X] = C[X^?](1)$ is the dual of Rickard's complex of ℓ-permutation modules.

Proof of 3.2. We can restrict to $N_G(H)$ if necessary, by naturality under inclusions, so we may assume that H is normal in G. Let $\mathcal{H}$ be the set of subgroups of G containing H and apply 3.3 to obtain $C[X^H](H) \cong L_{\mathcal{H}}C[X^?](H) = C[X^?](H)$.

Notice that $C[X^H](H) \cong C[X^H](1)$ by naturality under $G \to G/H$ and apply 3.4. $\square$

A similar method will prove the following result (see [9]). We abbreviate $H_0(G,M)$ to M_G.

Lemma 3.5 $C[X]_G \cong C[X/G]$.

Let $\mathcal{S}^1$ denote the set of non-trivial subgroups and write $S = S_{\mathcal{S}^1}$ and $L = L_{\mathcal{S}^1}$.

Lemma 3.6 $(C[X^?]/LC[X^?])^* \cong R\Gamma_c(X \smallsetminus SX, R)_0$.

Proof Both sides are zero on non-trivial subgroups, so we only need to check at the trivial group.

The inclusion map $C[SX] \to C[X]$ is equivalent to $LC[X] \to C[X]$. It is also dual to $R\Gamma_c(X,R) \to R\Gamma_c(SX,R)$. Thus the triangles $LC[X] \to C[X] \to C[X]/LC[X]$ and $R\Gamma_c(X \smallsetminus SX, R) \to R\Gamma_c(X,R) \to R\Gamma_c(SX,R)$ are dual. □

4 Smith Theory

Various results are known collectively as Smith Theory (see [4], for example), but the prototype is the theorem that if a p–group P acts on a finite dimensional CW–complex which has the mod–p cohomology of a point then the fixed point subcomplex also has the mod–p cohomology of a point. Once the case of P of order p is proved this follows by induction on the order of P.

From now on we will take R to be $\mathbb{F}_p$. We allow X to be either a CW–complex, in which case our results are well known, or a separated scheme of finite type over an algebraically closed field k. In the latter case the ℓ in the previous section becomes p and as a consequence we will need the characteristic of k not to be equal to p in order to be able to use the étale cohomology.

As before, we define $H^*_G(X,L) = H^*(\mathrm{Hom}_{\mathrm{CS}_R(G)}(C[X^?],L))$.

Lemma 4.1 *We have the following identifications:*

$$\begin{aligned} H^*_G(X,(RG)^?) &\cong H^*_c(X,R), \\ H^*_G(X,(RG^?)/(RG)_0) &\cong H^*_c(SX,R) \\ H^*_G(X,R_0) &\cong H^*_c((X \smallsetminus SX)/G,R). \end{aligned}$$

The analogous result for G–CW–complexes is well known.

Proof By the adjointness property of $(RG)^?$,

$$\begin{aligned}\mathrm{Hom}_{\mathrm{CS}_R(G)}(C[X^?],(RG)^?) &\cong \mathrm{Hom}_{RG}(C[X],RG)\\ &\cong \mathrm{Hom}_R(C[X],R)\\ &\cong R\Gamma_c(X,R).\end{aligned}$$

Notice that $C[SX] \cong LC[X]$, by 3.3. Because $LC[X]$ is in $\mathrm{add}(\{R[G/H^?]; H \neq 1\})$ we obtain

$$\begin{aligned}\mathrm{Hom}_{\mathrm{CS}_R(G)}(C[X^?],(RG)^?/(RG)_0) &\cong \mathrm{Hom}_{\mathrm{CS}_R(G)}(LC[X^?],(RG)^?)\\ &\cong \mathrm{Hom}_{RG}(LC[X],RG)\\ &\cong \mathrm{Hom}_{RG}(C[SX],RG)\\ &\cong \mathrm{Hom}_R(C[SX],R)\\ &\cong R\Gamma_c(SX,R).\end{aligned}$$

There are no non-zero homomorphisms from $\mathrm{add}(\{R[G/H^?]; H \neq 1\})$ to R_0. Also $C[X \smallsetminus SX]$ is in $\mathrm{add}(\{R[G^?]\})$, so vanishes off the trivial group. We find that

$$\begin{aligned}\mathrm{Hom}_{\mathrm{CS}_R(G)}(C[X^?],R_0) &\cong \mathrm{Hom}_{\mathrm{CS}_R(G)}(C[X^?]/LC[X^?],R_0)\\ &\cong \mathrm{Hom}_{\mathrm{CS}_R(G)}(C[(X \smallsetminus SX)^?],R_0)\\ &\cong \mathrm{Hom}_{RG}(C[X \smallsetminus SX],R)\\ &\cong R\Gamma_c((X \smallsetminus SX)/G,R),\end{aligned}$$

by 3.5. □

May's approach to Smith Theory considers the Bredon cohomology groups in the lemma above and uses various long exact sequences associated to a short exact sequence of coefficient systems.

Let I denote the augmentation ideal of RG. Notice that if G is a p–group, which we will denote by P, then $(RP)^?/I_0 \cong (RP)^?/(RP)_0 \oplus R_0$ and the composition factors of I_0 are all R_0.

Let

$$\begin{aligned}a_q &= \dim H^q_G(X,R_0) = \dim H^q_c((X \smallsetminus SX)/G,R),\\ b_q &= \dim H^q_G(X,(RG)^?) = \dim H^q_c(X,R),\\ c_q &= \dim H^q_G(X,(RG^?)/(RG)_0) = \dim H^q_c(SX,R).\end{aligned}$$

May proves the following result in [8] for P–CW–complexes but, since the proof uses only manipulations with Bredon cohomology and the identifications in 4.1, it is valid for separated schemes of finite type too.

Theorem 4.2 (Floyd, May) *The following inequality holds for any $q \geq 0$ and $r \geq 0$:*

$$a_q + \sum_{i=0}^{r} (|P|-1)^i c_{q+i} \leq \sum_{i=0}^{r} (|P|-1)^i b_{q+i} + (|P|-1)^{r+1} a_{q+r+1}.$$

In particular, if $a_i = 0$ for i sufficiently large,

$$a_q + \sum_{i \geq 0} (|P|-1)^i c_{q+i} \leq \sum_{i \geq 0} (|P|-1)^i b_{q+i}.$$

Moreover, if $a_i, b_i, c_i = 0$ for i sufficiently large then

$$\chi_c(X) = \chi_c(SX) + |P| \chi_c((X \smallsetminus SX)/P).$$

If P is cyclic of order p, and r is even if $p \neq 2$, then we can remove the factors $(|P|-1)$, i.e.

$$a_q + \sum_{i=0}^{r} c_{q+i} \leq \sum_{i=0}^{r} b_{q+i} + a_{q+r+1}.$$

Remark (1) If X is a CW–complex then we can use ordinary cohomology instead of compactly supported cohomology provided that we also replace $(X \smallsetminus SX)/G$ by $(X/SX)/G$ and take its reduced cohomology.

(2) Notice that the last line includes Illusie's result [6] for varieties that if P acts freely on X then $|P|$ divides $\chi_c(X)$. In fact, in this case, $C[X]$ is a complex of projective RP-modules and, since P is a p–group, the modules are free.

(3) In the topological case, if we take X to be EP (the universal cover of the classifying space) and $q = 0$ then we recover the well-known result that the $H^i(P, \mathbb{F}_p)$ are non-zero in every degree.

Recall that

$$H_c^i(\mathbb{A}^n(k), \mathbb{F}_p) = \begin{cases} \mathbb{F}_p, & i = n \\ 0, & \text{otherwise} \end{cases}$$

provided that p is not the characteristic of k.

Corollary 4.3 *Suppose that X has the cohomology of an affine space $\mathbb{A}^n$ and also that if X is a CW–complex then it is finite-dimensional. Then X^P has the cohomology of some affine space $\mathbb{A}^m$ for some m, with $n - m$ even if $|P| \neq 2$.*

Remark (1) By taking $n = 0$ this includes the case that when X is mod-p acyclic then X^P must also be mod-p acyclic.

(2) When X is compact a similar argument shows that if X is a mod-p homology sphere then so is X^P.

Proof By induction on P we can reduce to the case when P is cyclic of order p. For P must have a normal subgroup Q of index p, and by induction X^Q has the cohomology of an affine space. But $X^P = (X^Q)^P$.

From the last line in 4.2 with r large it follows that $\sum_{i\geq 0} c_i \leq 1$. The sum can not be 0 by the Euler characteristic formula. □

We now present a more conceptual approach to these results which shows how coefficient systems can provide a very flexible tool. It is based on the following lemma:

Lemma 4.4 *Any monomorphism between two projective coefficient systems in* $\mathrm{CS}_R(P)$ *is split.*

Proof Consider a map $R[P/U^?] \to R[P/V^?]$. It must be zero unless U is conjugate to a subgroup of V. But then it can only be a monomorphism if $|U| \geq |V|$ so in fact U is conjugate to V and the map is an isomorphism.

Now any projective F is of the form $F \cong \bigoplus_{j\in J} F_j$, where each F_j is an indecomposable projective, so isomorphic to some $R[P/V^?]$. So suppose that we have a monomorphism $f\colon\ R[P/U^?] \to \bigoplus_{j\in J} F_j$. The socle of $R[P/U^?]$ is just the sub-system generated by $\sum_{g\in P/U} gP$ in degree 0. One of the components of f, say $f_j\colon\ R[P/U^?] \to F_j$ must be non-zero on the socle, hence a monomorphism and so an isomorphism. The splitting is now projection onto F_j followed by $(f_j)^{-1}$.

Now consider the case $f\colon\ \bigoplus_{i\in I} E_i \to F$. If I is finite, say $I = \{1, \ldots, n\}$, then we have a proof by induction on n. We have shown that $F \cong E_1 \oplus F/f(E_1)$, and there is an injection $f'\colon\ \bigoplus_{i\in I\setminus\{1\}} E_i \to F/f(E_1)$. The latter splits by the induction hypothesis.

The case of finite I is enough for us to deduce that, for any I, the map f is pure. But, for modules over an Artin algebra, any projective module is pure injective (because it is a summand of a free module and the free module of rank 1 is Σ-pure-injective by condition (iii) of theorem 8.1 in [7]), so f is split. □

Corollary 4.5 *If C is a complex of projectives in* $\mathrm{CS}_R(P)$ *that is bounded above and such that $C(1)$ is exact then C is split exact.*

Proof Evaluation at 1 detects monomorphisms between projectives, so there is an easy argument based on 4.4 and induction on the number of boundary maps that can be split, starting from the left. □

Corollary 4.6 *Let $f\colon C \to D$ be a map between two bounded complexes of projective coefficient systems in $\mathrm{CS}_R(P)$. If $f(1)\colon C(1) \to D(1)$ is a quasi-isomorphism, i.e. induces an isomorphism in homology, then f is a homotopy equivalence.*

Proof Apply 4.5 to the cone of f, to deduce that f is a quasi-isomorphism. Since the complexes consist of projectives, f must be a homotopy equivalence. □

Corollary 4.7 *Let $f\colon X \to Y$ be a finite morphism of separated schemes that induces an isomorphism on étale cohomology with coefficients in R. Suppose that P acts on both X and Y and that f is equivariant. Then the induced morphism $f^P\colon X^P \to Y^P$ also induces an isomorphism on cohomology.*

Remark It is not sufficient to consider complexes of p-permutation modules. For example, if we let C_2 denote the cyclic group of order 2 take $R = \mathbb{F}_2$ then there is a short exact sequence $R \to RC_2 \to R$. But this is not split.

Remark The methods of equivariant cohomology of Borel [1] can also be applied to varieties. They all depend on analyzing the triangle $C[S_{\mathcal{A}}X] \to C[X] \to C[X]/L_{\mathcal{A}}C[X]$ of RG-modules. The proofs in [5] and [11] carry over.

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Received: 19 December 2002 Revised: 5 March 2004