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RATIONALITY, PROPERTY RIGHTS AND THERMODYNAMIC APPROACH TO THE MARKET EQUILIBRIUM

Abstract. We suggest a new approach to the description of complex economic systems. The main idea is to represent the phase space of the system by means of linear constraints on the differentials of the defining parameters of the system, i.e., by means of a system of Pfaff equations. Further investigation of the dynamical trajectories could be reduced to the studies of geometry of integral surfaces of the system. This approach assumes a non-conventional definition of the notion of economic equilibrium in terms of nonholonomic systems, more precisely, in terms of statistical thermodynamics.

In particular, our approach to economics explains the causes for unemployment and reveals mathematical reasons due to which "shock therapy" in economics of East European countries and Soviet Union in 1990s did not lead to the result promised to the peoples of these countries, in particular, why money fled out of these, poor, countries to the richer ones, and why during the same period China and Vietnam experienced an unusual economic growth.

Our approach makes manifest the reasons why honesty in market relations has a price and qualitatively evaluates it; we also indicated the limits of rationality of behavior of market's agents (buyers and sellers).

Economic equilibrium may be considered, citing A. Smith [1], through the metaphor of "invisible hand." At the time when A. Smith was working on his book, the principles of thermodynamics were not known yet, so the "invisible hand" was interpreted in terms of a mechanical metaphor. A. Smith's conceptual model that identifies human interests with "forces" of the market gives really serious grounds for such an interpretation.

Let us consider an "imaginary experiment" of A. Smith a bit closer. Smith assumes that an increase of commodity prices brings about an increase in at least one of the price-making components – the rent, workers' salaries or profit – giving a signal to actors, or rather forcing them, to change their behavior (to exploit more facilities, increase the offer of jobs or to expand manufacturing), and this change of behavior finally leads to the reduction of price.

There is, however, one very weak spot in the "imaginary experiment" of A. Smith. *The only immediately accessible information for the market actors is the price.* Whatever changes it – up or down – the reasons for this change are not immediately revealed to the observer. It is most often not clear to the buyer at all if the changes in the price were due to change-making factors (say, raise or fall of profit) or an increase of demand was the reason? Usually, such information is the seller's most guarded secret.

In the studies of researchers from the Austrian school we find serious arguments in favor of the hypothesis that *no complete information on the hidden components of the price is available for the market actors.* This, consequently, means that they simply cannot behave in the way described in A. Smith's "imaginary experiment." This necessitates in putting under doubt the mathematical metaphor underlying the classical concepts of the market dynamics, i.e., the mechanical metaphor.

In other words, the ontology of the classical model of economics is, indeed, not indisputable. F. Hayek [2] directly asserts that the market economy is based on the observance of moral principles, ensuring survival of the community in competition with the other communities, and that not infrequently these principles directly contradict mercantile egoistic interests.

M. Weber, in his famous work about the role of protestant ethics in genesis of capitalist economy, also developed similar views. Weber conjectured that *the interests of the members of society are being fulfilled* **as a result** of *abiding the moral laws in the society as a whole.* This directly contradicts the idea of A. Smith that *the common well being is a resulting corollary of individuals attempting to fulfill their egoistic interests.*

In neoclassical models of the economical equilibrium constructed later, such, for example, as the ones due to Arrow–Debreu–McKenzie, in order to prove the existence of equilibrium, one assumes that the subjects of economic activity maximize their utility functions under conditions of complete knowledge of the economic situation. Thus, the arguments on in-built incompleteness of the information available for the market actors were ignored.

This postulate – on unavoidable incompleteness of the information – seems to be one of the basic reasons why the economists of the Austrian school rejected applicability of mathematical methods in economics. It looks as though these scientists were not so much dissatisfied with the mathematics itself, but rather with the mechanical metaphor used to construct models of equilibrium – the metaphor in which, in the end, the utility played the same role as the potential in the classical dynamical systems.

Generally speaking, the violation of the postulate on availability of complete information about market situation destroys the very idea of rationality of market agents. How is it possible to be rational when you do not understand the situation?

H. Simon's idea of "bounded rationality" [3] emerged from that doubt and the development of institutional economics became unavoidable. Institutions, defined by D. North [4] as restrictions on human behavior produced by the fact that we are not able to rationally calculate everything, and therefore need certain rules to deal with our social environment, should be unseparable ingredient in the theory of market equilibrium. Nevertheless, this is very difficult to perform and up to now the problem how to take into account the existence of institutions within the framework of neoclassical economic theory is not solved.

Another difficulty for neoclassical theory is the problem of property rights. From the moment R. Coase [5, 6] showed that property rights are related with transaction costs and its implementation into theory produces important changes in economic equilibrium, the necessity to implement the institutional considerations into economic theory became obvious. Williamson [7], who showed that the vertical integration of firms is a result of minimization of transaction costs, developed his arguments still further.

In what follows we shall try to construct a theory of market equilibrium on the following assumptions which are exactly opposite the main assumptions of the neoclassical theory: We start with the implementation of the property rights and neglect rationality, i.e., consider systems with "zero intelligence" of the agents. It was shown recently that the assumption of "zero intelligence" is a good approximation in the analysis of some cases of financial markets[1]. Market consists of exchanges. Any exchange presupposes a kind of reflection. In fact, it presupposes a tower of reflections consisting of reasoning of the type "I know that he knows that I know ...." Any exchange is a kind of a game. But it seems that, to analyze reflective games, we could use a sequence of approximations that depend on the level of reflection. In that sense the "zero-level" approxi-

[1] I am very grateful to Doyne Farmer for intensive discussions of the subject at his seminar and private conversations during my stay at the Santa Fe Institute. He also suggested to use the term "zero intelligence" for systems described by physical statistics.

mation without reflections at all should also be valid and in some cases could give an important result.[2]

§1. The metaphors of equilibrium

The theory of economics is an example of a non-trivial case in which ontological assumptions that underlie mathematical models are mathematical models themselves or, to say more precisely, are mathematical metaphors. A basic mathematical metaphor for the classical model of the market is that of a *mechanical equilibrium*.

In some, very important, sense "the invisible hand" of the market in this model is equivalent to a mechanical force. The economics is considered as a dynamical system. *Time* stands as a key notion, and the mathematical structure of the economical models is represented by a system of differential equations.

In the mechanical models, the equilibrium is considered as a state at which the forces applied to the system counterbalance each other and the potential energy attains its extremum.[3] Consequently, to apply the mechanical metaphor of equilibrium in the economics, some analogs of the mechanical notions are needed. Such a conceptualization is not "harmless" at all: it implies that the system, having slightly digressed from the state of equilibrium, will return to this very state being left alone. Why should the equilibrium be stable?!

As far as the dynamics of the market is concerned, the neoclassical economics inherited the classical approach. Here lie the roots of ideas how to revitalize the economics via financial stabilization, the essence of monetarian approach to vitalization of economics. According to it, it suffices to release the prices while preserving the volume of money for the system to immediately come to an equilibrium.

The practice of "shock therapies" illustrates that this is not always the case. Still, practice may lead us astray. To unearth the reasons why the economical systems refuse to come to an equilibrium as predicted, we

[2] See, for example, the "TIT FOR TAT" strategy for Prisoner Dilemma game [8].

[3] It is worth remembering, among other things, that the subject of the modern research in mathematical economics is, as a rule, the state of equilibrium in itself. The dynamics of the system is only considered as a metaphor, pointing out the way this equalibrium state may be achieved. See [9, p. 3]. See also classical works such as [10–12]. In general, further evolution of the theory produced such a state of affairs in which the system's dynamics in the vicinity of the equilibrium point was completely neglected by the researchers.

have to deeply analyze, first at all, the "mathematical metaphor" used. The question is:

are the dynamical systems adequate and sufficient a metaphor to describe the equilibrium of economical systems?

In physics, as is well known, there are other, distinct, approaches to the conceptualization of the intuitive notion of equilibrium. **Our construction is based on the thermodynamic notion of equilibrium.**[4] According to this concept, the system gets in the state of equilibrium not because it is being affected by "forces," but simply because this is the most probable state of the system, consisting of numerous parts, each of which is characterized by its independent dynamics.

This approach may refer as well to mechanical systems obeying the laws of mechanics. But, if the system is very complex, its *general* behavior is determined by absolutely different principles, very unlike those of mechanics.

This distinction in the mathematical description of how the system changes its state is fundamental. In terms of thermodynamic approach to equilibrium, the system, instead of evolving in time, simply changes its position in the space of macroscopic parameters, remaining on a certain surface, the *surface of state*, singled out by the "equation of state."

Time is not included into the set of parameters important for the description of the system's equilibrium. The equation of state is given by (linear in the simplest approximation) dependencies between the differentials of macroscopic parameters, in other words, by a system of *Pfaff equations*[5]. By changing one or several macro-parameters of the system we simply move the system along the surface of state. It is important to notice that there exists a similar formulation of classical mechanics first given by É. Cartan [14] and developed by V. Arnold [17] where time doesn't play a distinctive role.

In essence, from the mathematical point of view, the investigation of equilibrium in such a model is a problem of differential topology of the

[4]For the first time the basic idea of this paper was presented at the conference "Economic development in East Asia" in Beijing, October 1986. Then it was presented at the NATO research workshop "Formal Descriptions of Developing Systems," October 2002, Honolulu, and during a number of seminars at the Santa Fe Institute. Further developments of this idea were gathered in my book [13] partly translated into English as a working paper of the Santa Fe Institute *Thermodynamic approach to the market equilibrium*, Santa Fe preprints, 2003.

[5]A *Pfaff equation* is an equation of the type $a = 0$, where a is a differential 1-form. For a modern exposition of the theory of Pfaff equations, see [14–16].

surface described by the equation of state.

Such a metaphor of equilibrium essentially differs from the mechanical one. Time does not occur here as an internal parameter of the system, the parameter that determines its dynamics, but as an external one.

Dependencies between the differentials of changeable macro-parameters are determined by the internal structure of the system, for example, by the energy values of subsystems.

In the beginning of the 20th century, C. Carathéodory proved [18, 19] that it is possible to logically develop the thermodynamic theory, drawing, exclusively, on the assumption that the "equation of state," i.e., a surface in the space of thermodynamic variables, corresponding to a system of Pfaff equations, exists. In this case, the second law of thermodynamics is formulated as existence, in an infinitesimal neighborhood of each state of the system, of such states that cannot be reached without the change of entropy.

In other words, a simple assumption that the differentials of "generalized positions" are constrained and this constraint may be non-integrable (in Hertz's words, *non-holonomic*[6]), appears to be sufficient to develop a system of thermodynamic equations. Accordingly, the idea of equilibrium will look completely differently. This thermodynamic equilibrium, unlike that from a mechanical metaphor, would mean not the existence of a singular point of a solution of a system of differential equations or an extremum of a potential function, but movement along the surface of state.

In what follows I will show that there are most serious grounds to believe that the Pfaff equations (and, consequently, thermodynamic metaphor of equilibrium) are often more adequate for the description of various economical phenomena than the mechanical metaphor.

Under some additional assumptions on the thermodynamic system described by the Pfaff equations, the *Le Chatelieau principle* [21] is applicable. Namely, the system demonstrates a behavior obstructing influences exercised on it as a result of changes of the external macro-parameters.

An important discovery of the past was that the phenomena of such homeostasis in physics, sometimes appearing as almost reasonable behavior, can be explained on the basis of the thermodynamic metaphor, proceeding from the very simple assumption, namely, that *for the most time the system is staying in the most probable state.*

[6] On non-holonomic systems, see [20].

The idea to use a thermodynamic approach in economics is not new[7]. Nevertheless, until now, the idea to apply the notion of entropy to the study of economics was only realized for particular purposes (mainly, in relation to the transportation problems, or to analyze the general features of economic equilibrium without attempts to analyze an equilibrium under institutional constraints). If we implement property rights, the model is changing radically. We obtain a possibility to calculate the equation of state for an economic system, taking into account institutional constraints.

The general idea of thermodynamic approach to analysis of economical equilibrium is as follows. If the system is described on two ontological levels – a "macroscopic" and a "microscopic" ones – and one macroscopic state is characterized by a multitude of microscopic states defined as a distribution of property among economic agents (the number of states is called the *statistical weight* of the macroscopic state), and the system will, generally, remain in the most probable state, i.e., in the state with the greatest statistical weight.

Therefore economics is a theory of distributions of property rights.

At the same time we do not assume that the changes of property rights or economic exchanges (which are, in general, the exchanges of properties) are guided by rational reasons. **Instead, we champion the notion of equilibrium: it is the equilibrium that guides the exchanges and imposes in reality very strong restrictions on the processes of exchange.**

The conditions of applicability of the thermodynamic approach can be formulated in very general terms. It becomes clear thereby, that the applicability of thermodynamics goes beyond the realm of physical systems. One can imagine a Large System that can be decomposed into,

[7]In late 1960s, Wilson [22] suggested that the entropy analysis should be used for the examination of transport flows. Wilson mainly used the method of the maximization of entropy to tackle transportation problems. In 1973, L. I. Rosenoer published three papers on thermodynamic approach to control processes [23], and later three papers with A. M. Tsirlin [24]. In these papers an idea was suggested: to take into account a parallel between entropy and utility function and to use the thermodynamic equation of state for control actions.
The use of entropy approach is also well-known in the economic studies in relation to the estimation of incomplete data on the basis of the ideas by E. T. Jaynes [25, 26]. For details, see [27, 28]. These works do not include, however, analysis of economic equilibrium. On the precedents of application of other methods from statistical physics for socio-economic studies, see, for instance, [29, 30].

or rather, which consists of, a huge number of Small Systems, each with its independent dynamics. We assume that the state of the Large System, and its relatively large parts, is described by a certain number of macro-parameters, which are additive, i.e., *the total values of the macro-parameters appear to be the sums of values of the same macro-parameters of parts.*

In physics, *energy* is an example of such a parameter if we neglect superficial interaction between the parts of the Large System. This is a basis for application of thermodynamic methods in physics. Let further each of the systems considered be additionally characterized by a set of micro-parameters which can have distinct values at the same value of the fixed macro-parameter. Their values are determined by the system's dynamics and, generally, can be of interest in relation with our task in one aspect only. Namely, having fixed them, an exact state of the system becomes known and one can answer how many various micro-states correspond to one macro-state of the system.

At this stage, we can introduce the notion of statistical weight, as a number of micro-states corresponding to one macro-state, and the notion of entropy, as the measure of uncertainty of the macro-state of the system, which is a function of the number of micro-states.

If the system is such that the micro-states of the parts of the Large System are statistically independent, it is possible to compute the statistical weight of the Large System as a whole, given the statistical weights of all Small systems. For that purpose, it suffices to multiply the statistical weights of Small Parts.

If we wish the entropy be additive, we may regard it as the logarithm of the statistical weight. Since $\ln(ab) = \ln a + \ln b$, it follows that the uncertainty, or entropy, is additive. In fact, the statistical weight can be calculated as a functional integral over field parameters (values of this parameter for the parts of the system) under the conditions of conservation of a sum of these field parameters.

Assuming that nothing is known about the dynamics of the system, except that it is very complex, the natural assumption for the probability value of any macro-parameter is that it is proportional to the number of the appropriate micro-states, i.e., to the statistical weight. Since the logarithm is a monotonous function, the most probable state, i.e., the state with the greatest statistical weight, is, at the same time, the state with the greatest entropy, i.e., with the greatest extent of uncertainty.

This is the core of the *second law of thermodynamics – the entropy* *tends to increase* – as every part of the Large System goes in the most probable state during interactions with the other parts.

If we "isolate" some part of the Large System in order to observe the distribution of the probable states, we have to consider the rest of the Large System as a thermostat, that is a reservoir that ensures equilibrium (in a thermodynamic sense) of the distribution of states within the subsystem isolated.

To find out the form of this distribution, it is necessary to introduce the notion of "temperature," equal to the inverse of the derivative of entropy of the Large System with respect to the macro-parameter for which there exists a conservation law.

The introduction of the parameter of equilibrium – temperature – is simply a result of the condition that *there are no flows of the conserved macro-parameter between the parts of the system.* If there are several conserved macro-parameters, then there are as many parameters of equilibrium as there are conservation laws.

Observe that we said nothing related to either physics, or physical laws. All the arguments are applicable to Large Systems of any nature, subject to the above hypotheses. These arguments look rather natural, in relation to the large economic systems, if we regard the total income, the total value of products or the total value of consumption of goods as macro-parameters, whereas distribution of property rights on income and products of consumption of goods between the subjects of economic activity are viewed as micro-parameters.

In this approach to the study of economic equilibrium, we avoid the necessity to explore the subsystem's dynamics, so the knowledge of institutional limitations on the goods production and distribution suffices. Thus, we offer

not only a new approach, appropriate to describe the economic equilibrium, but, also, the instruments to investigate the impact of institutional restrictions on the state of equilibrium.

Namely, we get the opportunity to build up a mathematical apparatus for analysis of the transaction costs theory, to suggest quantitative methods of the study of the impact of the information asymmetry on the behavior of the market's agents and to tackle many other problems as well.

If the macro-parameters are functionally dependent, and the surface of state is differentiable, then the differentials of the macro-parameter are related by a system of Pfaff equations.

Thus, the idea of thermodynamic equilibrium is quite appropriate for description of economical systems. They have observable flows of money, goods and people, and, like physical systems, have two levels of description.

Consequently, the description of the economic system should be equivalent to the description of physical systems in thermodynamics, though the parameters of equilibrium – "temperature," "pressure," "chemical potential" – will certainly have quite different interpretations, the ones that mirror the particularities of economical systems.

In fact, we may consider a general system described by parameters $x, \dots, z$. If we change the state of this system, small changes may be considered as differentials $dx, \dots, dz$. Generally speaking, if the system is constrained by internal links, the differentials of parameters are not independent. They are related by linear equations of the form

$$A(x, \dots, z)dx + B(x, \dots, z)dy + \dots = 0.$$

We can find a number of such Pffaf equations that single out our system.

Such a general case is too complicated for the analysis. Let us restrict ourselves with a simpler case with one Pfaff equation and a number of functional dependencies between parameters of our system:

$$f(x, \dots, z) = 0, \quad g(x, \dots, z) = 0, \dots .$$

We can change the system of coordinates and minimize the number of independent parameters of the Pfaff equation. The smallest such number is called the *class* of the Pfaff equation. The class of one equation is always odd $(2k + 1)$ [15].

We have just obtained a system completely identical to the generalized Hamiltonian system studied by Dirac [31] (see also Faddeev [32], Pavlov [33]). Formally, all functional constraints are equivalent and each one may be considered as a Hamiltonian function.

In the analysis of generalized Hamiltonian system, time plays an outstanding role. In our case, we do not need to distinguish the role of one parameter. We may choose one constraint $f(x, \dots, z) = 0$ and define the characteristic direction of our Pfaff equation on the surface defined by constraint, mentioned above (see [15, Chap. IX]). It is possible to relate a vector field to each field of characteristic directions. We may act by this vector field on any function $g(x, \dots, z)$. The result is called the *Jacobi* or *contact* bracket $[f, g]$ (see [14, Chap. IX]).

The difference between the Jacobi and Poisson brackets consist of the fact that the Poisson brackets are defined on an even-dimensional space with a bivector whose Schouten bracket with itself vanishes (if the bivector is nondegenerate, it is equivalent to a nondegenerate closed differential 2-form that determines a symplectic space). The Jacobi brackets are defined not by a bivector on a space of even dimension, but by a contact form (in a space of odd dimension).

To find a k-dimensional integral surface on the surface singled out by the equations of functional constraints is only possible if all Jacobi brackets between the constraints vanish on this surface [15, Chap. IX]. If some of these brackets do not vanish we must include the result of bracketing into the set of constraints and calculate Jacobi brackets between all the elements of the new set. After several iterations we may come to one and only one of the following results:

(a) The number of constraints exceeds $k + 1$, so no integral surface of the Pfaff equation exists.

(b) The process terminates and we get a surface on which we can find an integral surface of our Pfaff equation. In absence of singularities this surface simply consists of k-dimensional integral surfaces of the Pfaff equation.

Despite the simplicity of the argument the consequences could be very important for the economic systems. **We need a check of coherence of constraints imposed on the system**. This means that, in the case of economic system, control actions cannot be arbitrary. Any additional constraint imposed on the system must be checked on compatibility with other constraints, it should not destroy the very existence of a solution of the system of equations.

Consider a hypothetical example. Imagine that, in a certain country, the currency board is established. This means that an additional link is created between the country's economy and the world economy. Could we be sure that such an additional link would not destroy the equilibrium of the country's economy?

One more remark. It is known that, for some equations of generalized Hamiltonian systems, there exist certain hidden, "non-physical" variables [34]. This means that, under certain conditions, there exist functions of parameters of the system that could be changed without visible effect. A contemporary physical theory (gauge theory) attributes a great importance to the role of such cases. It is rather interesting to investigate what kind of effects could be connected with that type of equations. We may conclude here with one very general and challenging statement: The integral surface of a system of Pfaffian equations has some geometric features. It is possible to define, for example, different connections on this manifold, using the natural frame bundle, or to define characteristic classes. All such geometric objects may be interpreted as dynamic objects, if dynamics is introduced properly. Therefore we may change the direction of inference and interpret equilibrium economical dynamics as geometry.

Of course, this is only general definition of a very complex program of future studies, but I am convinced that the realization of such a program could change significantly our understanding of economic processes.

§2. The entropy and temperature in models of economics

We begin our description of a thermodynamic model of economics with the simplest example. Let the economic system consist of N agents, among whom the income, constant for the system as a whole, is distributed. We assume that there are many ways to distribute income, and we are incapable to foresee all possible alternatives. This assumption fully corresponds to Hayek's concept of market in which the market is described as an arena of discoveries of new procedures and operations. For ultimate simplicity, we assume that the income is quantum, i.e., presented in integers (which is natural, as the smallest unit of currency is operational in economics).

Now, for each value of the total income E, it is possible to find the quantity of modes of income distribution between the agents, as a characteristic $n(E, N)$ of this value, called the *statistical weight of the state* with income E.

At this stage, we can introduce the concept of equilibrium. The idea is that two systems (under the above hypothesis) are in *equilibrium* if the distribution function of income does not change when they enter in a contact, hence, there is no income flows between the systems. By a "contact" we understand here an "open list" of possible modes of redistribution. It is remarkable that it is possible to calculate the statistical weight regardless of uncountable variety of various institutional limitations imposed on the agents' incomes, so the functions $n(E, N)$ may be different for different systems.

If two systems interact, one with the total income E_1 and the number of agents N_1, and another one, with E_2 and N_2, respectively, then the total system is characterized by the total income $E_1 + E_2$ and the number of agents equal to $N_1 + N_2$. We assume also that *all elementary states of* *income distribution have the same probability.*

The main ground for such an assumption is symmetry of states. As in the probability theory and in statistics, we assume equal probability of elementary events just because there are no grounds to prefer one event to another. Thus, it is of utmost importance for the theory to consider all probable states of distribution. The change of function $n(E, N)$ will, certainly, change the results obtained in this model.

To find the state of equilibrium is to determine conditions under which the income is not to be redistributed between the interacting systems. For this purpose, consider a redistribution of income during an interaction. Let a certain part of income, ΔE, pass from system 1 to system 2. Then, the states of the systems change and their statistical weights become equal to $n_1(E_1 - \Delta E, N_1)$ and $n_2(E_2 + \Delta E, N_2)$, respectively.

The principle of equal probability implies that the most probable state of the integrated system is the one with the greatest statistical weight. So we should seek the maximum of function $n_{\text{tot}}(E_1, E_2, N_1, N_2)$, with the proviso that the total income $E_1 + E_2$ is a constant. If no transfer of agents from one system to another is possible, then the statistical weight of the integrated system is equal to

$$N_{\text{tot}}(E_1, E_2, N_1, N_2) = n_1(E_1, N_1) n_2(E_2, N_2). \tag{2.1}$$

Since $E_1 + E_2 = \text{const}$, it follows that $\Delta E_1 = -\Delta E_2$.

Instead of seeking the maximum of n_{tot}, we can seek the maximum of $\ln n_{\text{tot}}$, because the logarithm is a monotonous function. From $\ln n_{\text{tot}} = \ln n_1 + \ln n_2$ we derive the condition on the maximum. It is very simple:

$$\frac{\partial}{\partial E_1} \ln n_1(E_1, N_1) = -\frac{\partial}{\partial E_2} \ln n_2(E - E_1, N_2), \tag{2.2}$$

or, as $dE_1 = -dE_2$, we have

$$\frac{\partial}{\partial E_1} \ln n_1(E_1, N_1) = \frac{\partial}{\partial E_2} \ln n_2(E_2, N_2). \tag{2.3}$$

So, the two systems are in equilibrium if they are characterized by the same value of the parameter $\frac{\partial}{\partial E} \ln n(E, N)$.

In thermodynamics, the logarithm of the statistical weight is called the *entropy* (of the system), and its derivative with respect to energy is the inverse temperature,

$$\frac{\partial}{\partial E} \ln n(E, N) = \frac{1}{T}. \tag{2.4}$$

In order to reach the state of equilibrium, the interacting systems should be at the same temperature.

The economical system is in the state of equilibrium if it is rather homogeneous and there are no income flows from one of its parts to another. We certainly assume that the homogeneity is kept only unless there is no division into parts so tiny that significant income flows are observed.

We see that in our thermodynamic model, there are two extremely important characteristics – the entropy and temperature. If we do not know these parameters, we cannot correctly infer the conditions for the system's state of equilibrium: the system is in the state of equilibrium only when its subsystems have an identical temperature, and the temperature cannot be calculated without knowledge of entropy.

The basic possible objection against the thermodynamic approach to economy is that the number of "particles" involved (in the given example – the number of the market agents) is tiny as compared with the numbers of particles in the usually considered physical systems. In the physical systems, the number of particles is comparable, as a rule, with the Avogadro number, whereas in the economic systems it is usually about 10^3 to 10^8.

In statistics, the order of dispersion is equal to $\frac{1}{\sqrt{N}}$, where N is the number of particles in the system. Thus, in physical problems, if the statistical errors related with the fact that the number of particles is finite are completely insignificant due to the enormity of their number, in economical problems, we should expect much larger errors, up to 3%. Such errors do not look too large, actually, taking into account the extreme roughness of economical models.

§3. The thermostat and the function of income distribution

Now we will investigate how the income is distributed among the subjects of economic activity. Within the framework of thermodynamic model, it appears that there exists a universal function of distribution, whose configuration, if the number of the subjects is a constant, depends only on the temperature. This situation is well known in statistical thermodynamics, where such distribution is called the *Boltzmann distribution*.

To examine this distribution, imagine that the economic system X is very large, and, out of it, a small part, Y, is separated. Naturally, the system X is considered as a reservoir. If we consider a certain state of the small subsystem with income E_1, the probability for the small subsystem to have income E_1 is proportional to the number of possible states of the reservoir with the range of income $E - E_1$, under the hypothesis that the total income of the system is constant. Then the ratio of probability $P(E_1)$ for the subsystem to have income E_1, to probability to have income E_2, is equal to the corresponding probability ratio for the reservoir:

$$\frac{P_1(E_1)}{P_2(E_2)} = \frac{n(E - E_1, N)}{n(E - E_2, N)}. \tag{3.1}$$

Since $n = e^{S(E,N)}$, where $S(E, N)$ denotes the entropy of the system, we can express this ratio as

$$\frac{P(E_1)}{P(E_2)} = e^{S(E-E_1,N)-S(E-E_2,N)} = e^{\Delta S}. \tag{3.2}$$

If the reservoir is far greater than the subsystem in question, we can make expansion of ΔS into Taylor series in ΔE, thus obtaining in the first order $\Delta S \cong \frac{E_1 - E_2}{T}$, where T is the temperature of the reservoir, see (2.4).

It follows that

$$\frac{P_1(E_1)}{P_2(E_2)} = e^{-\frac{E_1 - E_2}{T}}. \tag{3.3}$$

This is an approximate formula for the probability distribution for a small system.

If the number of probable states of subsystem Y with incomes E_1 and E_2 is equal to $n_Y(E_1)$ and $n_Y(E_2)$, respectively, then the distribution formula takes the form

$$\frac{W_Y(E_1)}{W_Y(E_2)} = \frac{n_Y(E_1)}{n_Y(E_2)} e^{-\frac{E_1 - E_2}{T}}. \tag{3.4}$$

In this formula we proceed from the probability of the state, P, to the probability of the level of income, W.

The arguments laid out above are, actually, a replica of the traditional deduction of the Boltzmann distribution in statistical thermodynamics. With the help of the Boltzmann distribution function, we easily calculate the average income of one agent:

$$\overline{E} = \frac{\int_0^\infty E e^{-\frac{E}{T}} dE}{\int_0^\infty e^{-\frac{E}{T}} dE} = T. \tag{3.5}$$

We see that in absence of any restriction on the agent's income, the mean income is equal to the temperature. As we will see, under restrictions on income, the relationship between the temperature and the average income may look quite different.

In conclusion of this section, observe that, like in statistical thermodynamics, in the thermodynamic model of economics, the parameter called the *statistical sum*,

$$Z_0 = \sum_E n(E, N) e^{-\frac{E}{T}}, \tag{3.6}$$

is extremely useful. The sum here runs over all possible values of E. The probability of the system interacting with thermostat with temperature T can be expressed as

$$p(E) = \frac{e^{-\frac{E}{T}}}{Z_0}. \tag{3.7}$$

With a fixed number of agents, the average income in the system at temperature T is given by the formula

$$\overline{E} = \frac{\sum\limits_E E n(E, N) e^{-\frac{E}{T}}}{Z_0} = T^2 \frac{\partial}{\partial T} \ln Z. \tag{3.8}$$

§4. On interaction of systems with and without restrictions on income

In this section a "spin model," with two possible values of income, 0 and A, will be examined in more detail. Despite its seemingly too abstract nature, this model is rather useful, as it catches some important features of systems with restrictions on income. What might be an outcome once such a spin system enters into interaction with a "free" market system, the one without any restrictions on income?

In the free market system, the entropy increases together with the increase of energy, and the temperature is always positive. This is not always the case for system with restrictions on income.

What is the meaning of the *negative temperature*?

The phenomenon was investigated quite well in the laser theory [35]. The negative temperature implies the inverted population, i.e., a situation, in which the levels with higher energy are more densely populated than the ones with lower energy. In such a state, the system is imminently ready to ejaculate its energy at the contact with any system with positive temperature.

How can this be interpreted in terms of the income distribution described above? If two systems with positive temperature are in contact, the redistribution of income goes from the system with higher temperature to the one with the lower one. As we saw, for the free market, the temperature is equal to the average income per agent. The redistribution of income from the "richer" system to the "poorer" one takes place in accordance with our intuitive understanding.

Contrariwise, if a free market system interacts with a system with restriction on income, then, under certain conditions, a counter-intuitive process is observed, when redistribution of the income goes from the "poorer" system to the "richer" one.

Let X be a free market system, with the temperature T_X equal to the mean income per market agent. Let Y is a spin model with income bounded from above by A. Let $T_X > NA$ and let the number of Y's subjects with non-zero income exceed the half of their total number, so the temperature of Y is, evidently, negative.

What is going to happen when these systems start to interact? The entropy of the combined system will increase, and the entropy of system Y **will also increase** simultaneously, thereby precipitating disorder of system Y. Therefore, Y **must** transfer a part of its income to X despite the fact that average income of the system X is already higher than the average income in Y.

As a result, system Y, with restriction on income, will get still poorer, while system X, without any restriction on income, will get richer and richer.

This redistribution of income will not be an effect of coercion or looting, but just a consequence of the fact that the combined system tends to acquire the most probable state. As for coercion, it may play a certain role, but not in the form of X raping Y, but coercion inside system Y, aimed at obstructing any rise of income above certain level.

Applying the above most simple model to real economic situations, we can draw two important conclusions:

1) the policy of restriction of income is dangerous if the contact with a free market surroundings is unavoidable;

2) if such a policy was already embarked on, and the economy in which it was implemented is isolated from the free market, then the effect of the "market reforms" will depend on the sequence of two main steps, the release of incomes and the establishment of contact with the free market surroundings.

For the country with a "non-market" economy, it would be necessary **first** to release the income from restrictions and **then** "open" the economy to the market **only after the equilibrium had been achieved**. Otherwise, the resources of the country will be "sucked out" before the equilibrium is established.

Of course, our arguments are based on a very simple idealized model. Still, our reasoning gives an explanation of different results observed at carrying out market reforms in the Eastern Europe, on the one hand, and in China and Vietnam, on the other.

In the Eastern Europe, the reforms were conducted by means of a "shock therapy": the national economies were "opened up" for outside activities without preliminary creation of market institutions inside the country. The results were catastrophic (flight of capital and collapse of production). In China and Vietnam the reverse order was implemented: first, the release of capitals inside the country and then the market was gradually opened up for outsiders, in accordance, more or less, with the process of creation of a domestic market. Such a policy resulted in an astoundingly rapid economic growth.

Consider another case where a system with a restriction on income interacts with a free market system. Let the "spin system," a community of N workers, with fixed individual income A, have L vacancies. Obviously, for A fixed, the system's temperature is negative for $L > \frac{N}{2}$. This means that, *in the state of equilibrium, the free market would prefer that half of the workers should not get salary*, i.e., should be unemployed, because this is a far more probable state than the other ones with a lower rate of employment. So, this model includes unemployment as an inherent characteristic of the system in the state of equilibrium.

For the neoclassical school of economics, the following famous paradox stood as a stumbling block: why the market equilibrium is not realized when the free labor force market is in operation[8]? In the above model this paradox is dismissed. A negative temperature may also emerge in the system with restrictions on the salary *from above*. Indeed, to increase the total value of wages in the situation of a salary-restricted economy with the value of salary bounded from above by A, *means* that at the total value of wages E equals NA the entropy vanishes. This implies that, at

[8] The problem of unemployment is of utmost importance for the discourse on economic balance, as it represents by itself a "counter-example" for neoclassical theory of balance, stating that no excess demand is possible. This served as one of the greatest incentives for the emergence of institutional approach to the economy. See [7, 36].

least for some interval $E_0 < E < NA$, the temperature, $T = \frac{1}{\partial S/\partial E}$, is negative, i.e., at any contact with a free market the value of total wages will fall, at least to E_0.

If, moreover, there is a *minimal wage value*, B, the number of gainfully employed will be bounded from above:

$$L < \frac{E_0}{B}.$$

This restriction will keep salary from dropping down, and necessitate a certain rate of unemployment, which will be $\geqslant \alpha = \frac{N-L}{N}$. In reality, the level of wages is always limited "from below" by the level of biological survival (at least, in towns, where Nature's resources are unattainable). Therefore, if salary is bounded, the opportunity for unemployment is created.

The analysis given above shows that **unemployment emerges in the state of equilibrium** at the positive temperature if the wages are bounded simultaneously from above and below.

The above can be considered as a proof of Keynesian arguments on the reasons of unemployment. In real situations, the administration aspires to put a limit on the salary "from above," while the trade unions (are supposed to) do it "from below." This combination **must**, according to our model, result in the emergence of unemployment.

§5. The thermodynamics of the prices: a description of the problem

Let us try now to extend the thermodynamic approach to the study of the markets and prices. For this purpose, consider a simplified model situation.

The market is characterized by the presence of goods which are sold, and money which are spent at purchases. Let $V(t)$ be the amount of "units of goods" sold per unit of time during which the buyers spend $E(t)$ of units of money. We say that the market is *stationary* if $V(t)$ and $E(t)$ are constants independent of time and all goods are bought, i.e., there is no accumulation of goods in the hands of the sellers. While the market functions, the deals are made, i.e., agreements on exchange of some of the goods for some money.

Consider a model with several co-existing markets able to interact: exchange goods and money resources. The intuitive concept of equilibrium of these markets is that the situation in each of these markets remains, in certain essential aspects, the same even after the interaction. Clearly, this cannot take place for any value of the amounts of goods and money V_1, E_1 and V_2, E_2, respectively, in each system. Besides, the interaction can vary, i.e., include an exchange only of money resources, only of goods, or both.

Consider the simplest case, when the values of money and goods are discrete, as is the case in real life:

$$E_n = nE_0, \qquad V_m = mW. \tag{5.1}$$

In addition, let the values of n and m be sufficiently large to make it possible to treat small changes of flows as insignificant and differentiate. Let N be the number of buyers in our model. (Generally speaking, N should be the number of bargains, but it is more convenient to consider N as the number of buyers.) The market is assumed to be stationary in the above sense.

The set of probable states of the market is the set of all contracts allowed by rules, i.e., the ways of distribution of the goods between buyers. Let us examine the market with a single seller assuming that the seller is not capable to influence the flow of arriving goods.

For this model, introduce the entropy in the same way as earlier, namely, as the logarithm of the number of probable states. In this case, the entropy depends on both the total money expenditure, E, and the total amount of purchased goods, V. Various states of the market can be regarded as legalized by the rules of distribution of the total supply of money and goods among N buyers.

By analogy with the pressure in statistical thermodynamics, the notion of *marginal price* P is introduced as follows:

$$P = T\frac{\partial S}{\partial V}. \tag{5.2}$$

The marginal price is a characteristic that points out whether or not the two interacting systems are in the equilibrium.

First, consider the simplest case, referred to as the "free market." In this model there are no restrictions on goods and money distributions among the buyers. This means that the buyer can pay any price – either infinitesimally low or infinitely high.

The entropy of the system (the logarithm of the statistical weight) consists of the two components: one, is determined by the flow of goods, the another one, by the flow of money:

$$S(E,V) = S(E) + S(V). \tag{5.3}$$

First, compute the number of probable distributions of goods and money flows among N agents of the market. The statistical weight $g(E_n, N)$ of the flow E_n distributed among N agents is equal to the number of non-negative solutions of the equation

$$n = x_1 + \ldots + x_N. \tag{5.4}$$

Having performed calculation we get

$$S(E_n) \ln g(E_n, N) \approx$$

$$(N+n-1)\ln(N+n-\frac{1}{2})-(N-1)\ln(N-\frac{1}{2})m\ln(n+\frac{1}{2})-\frac{1}{2}\ln 2\pi. \tag{5.5}$$

The temperature T is expressed in terms of $\frac{\partial S}{\partial E}$:

$$\frac{1}{T} = \frac{1}{\varepsilon}\frac{\partial S(E_n)}{\partial n} \approx \frac{1}{\varepsilon}\ln\frac{N+n-\frac{1}{2}}{n+\frac{1}{2}} \tag{5.6}$$

For $n >> N$, which is a rather natural condition for the free market, the expression for the inverse temperature is as follows:

$$\frac{1}{T} = \frac{\partial S(E_n)}{\partial n} = \frac{1}{\varepsilon}\frac{N}{n} = \frac{N}{E}. \tag{5.7}$$

This means that the temperature $T = \frac{E}{N}$ is equal to the mean value of income per capita.

To compute $\frac{\partial S}{\partial V}$ is a completely analogous matter:

$$\frac{P}{T} = \frac{\partial S(V_m)}{\partial V_m} = \frac{1}{W}\frac{N}{m} = \frac{N}{V}. \tag{5.8}$$

Thus, a relation between the marginal price, the goods flow and the “temperature” of the free market is of the form

$$P = T\frac{N}{V}. \tag{5.9}$$

As $TN = E$, it is easy to show that for the free market the *marginal price is equal to a median price*. This, actually, allows to define the *price* as $T\frac{\partial S}{\partial V}$.

We can express the conditions of equilibrium as follows:

$$\left.\frac{\partial S_1}{\partial E_1}\right|_{V_1} = \left.\frac{\partial S_2}{\partial E_2}\right|_{V_2}, \quad \left.\frac{\partial S_1}{\partial V_1}\right|_{E_1} = \left.\frac{\partial S_2}{\partial V_2}\right|_{E_2}. \tag{5.10}$$

The first expression, as we already discussed, means the coincidence of "temperatures." The second one means coincidence of the values of marginal prices. Thus, coincidence of the values of marginal prices is a necessary condition for the interacting markets to be in the state of equilibrium.

Now, proceed to the case that the market is not "free"; let some restrictions be imposed on its operation. Let us investigate, first, what is going to happen if something similar to the "allocation system" is adopted, i.e., the possibilities to buy goods are forcefully restricted. Suppose that none of the market agents is allowed to get more goods than a certain amount, K. In all the other respects the distribution remains free, and – this is of utter importance – *it does not depend on the money distribution*, i.e., there are no explicit restrictions on the prices. This means that the statistical weight of the state with V goods and E money is still the product of statistical weights:

$$g_N(E, N) = g_N(E) g_N(V),$$

while the entropy includes two components, one of which depends on E, and the other one on V:

$$S_N(E, V) = S_N(E) + S_N(V). \tag{5.11}$$

To determine the marginal price, we should know $\frac{\partial S}{\partial V}\big|_{E,N}$, i.e., we should compute $S_N(V)$. This appears to be a difficult combinatorial problem: to find the number of non-negative integer solutions of the equation

$$m = x_1 + x_2 K - x_n, \quad \text{where } x_i \leqslant K \quad \text{for each} \quad i. \tag{5.12}$$

It is clear that, if $m > KN$, this problem has no solutions at all. The logarithm of the number of solutions is the system's entropy $S_N^K(V_m)$. This means that as m (i.e., the flow of goods) approaches KN from below, the entropy begins to decrease. But this, in turn, means that $\frac{\partial S(V_m)}{\partial V}$ becomes negative. Taking into account that the distribution of money flow is free, which implies positive temperature, the result would be that, under restrictions, the system's marginal price becomes negative when the volume of the flow of goods becomes sufficiently large.

The same picture with negative temperature exists for the restrictions of prices from below. This means that during interaction with other systems with positive marginal price, our "allocation system" with imposed restriction on consumption of the goods will begin to eject goods at dumping prices: any more-than-zero price of the exported goods will contribute to stabilization of the equilibrium. Of course, it is possible to say that when the flow of goods is close to the value KN, there is no sense to restrict the distribution. The fact, however, is that the marginal price can become negative *long before* the flow of goods approaches the critical value, KN.

As a byproduct of our arguments, we see that **introducing restrictions in the system we provoke occurrences of zones of latent instability**. The borders of these zones are extremely difficult to discern even for the simplest models.

Thus, restrictions on economic activity can, by no means, be held as "harmless." First of all, **no restriction is harmless because the range of their influence is unclear, the range within which their influence renders the system** ***unstable.***

Observe that the so-called "economic" reasons for various restrictions on deals (e.g., the minimal price determined by the cost of production) turn out, at deeper scrutiny, purely political reasons: the cost of production often can not be lowered "thanks" to a monopoly, a political control of the market.

The above study of the market with restrictions on prices, is related to the problem which lately aroused considerable interest in theoretical economics. In 1970, G. Akerloff [35] showed that, for asymmetric markets, **the presence of low quality goods and dishonest sellers can result not only in sweeping away of high quality goods from the market, but may also result in a collapse of the market as such**.

We interpret Akerloff's result not so much as a proof of the possibility for asymmetric markets to collapse, but as impossibility to apply to them the standard equilibrium techniques used in neoclassical analysis.

Indeed, consider two markets in interaction, one, with restrictions on prices from below, and another one, without such restrictions (say, a market of used cars). Consider a model with an "asymmetric information." We see that the market with restrictions on price from below is a market of "good" cars, while the market without restrictions is a market with defective cars. As we have shown above, at a certain value of parameters of the flow of goods and "temperature" of the market (namely, at low temperature), the marginal price in the market with restrictions (here: in the market of "good" cars) becomes negative. This means that this market is collapsing. But **the actual collapse will only happen at a rather low temperature**. At higher temperature the market of high quality cars is quite viable.

Thus, it is possible to make some amendments to Akerloff's statements on the market of "lemons," and on the opportunities to operate honest business in the countries, where "standards of honesty" are low.

Next, consider in more detail the thermodynamic model of the market with prices bounded from below. For this purpose, we apply the technique of the large statistical sum. If we forget about the sellers and buyers, and only consider the amount of contracts, N, then the temperature of the market, according to our previous consideration, can be defined as

$$T = \frac{E}{N}, \tag{5.13}$$

where E is the amount of money. Introduce the concept of a "potential contract" which means the possibility of striking a deal, with the goods flow $V_l = lV_0$, and money flow $E_k = kE_0$, where, respectively, V_0 and E_0 are the least values of flows. Then some further simplification would be expedient. Introducing discrete time labeled by moments T_k, we assume that the goods and money flows only occur at discrete time moments. Bearing in mind the statistical aspect of the model, it looks quite natural to consider the market as the set of simultaneously coexisting systems covering all admissible deals, and to carry out averaging over the ensemble instead of averaging over time.

The potential contract $C(E_k, V_l)$ with parameters V_l, E_k may be "filled in" with a real content, but also may remain "empty." The notion of potential contract corresponds to the notion of an orbital in quantum mechanics. We are able now to refer to the set of potential contracts, $C = (E_k, V_l)$ subject to some restriction, the contracts with $\frac{E_k}{V_\lambda} < Q$ may be ruled out (which, actually, mirrors the restriction on prices from below, Q being the least allowed price). Now, apply the standard technique of the large statistical sum. The equation of state takes the form:

$$\frac{P}{T} = \frac{NQ}{E_0} \frac{1}{e^{\frac{QV}{E_0}} - 1}. \tag{5.14}$$

The equation of state (5.14) is very interesting. When $Q << \frac{T}{V}$, equation (5.14) turns into the familiar equation of the ideal gas:

$$\frac{P}{T} = \frac{N}{V}. \tag{5.15}$$

But, as it turns out, even small restriction on price from below completely transforms the equation of state.

In the range of larger V, the marginal price P (i.e., the equilibrium price) becomes smaller than the minimal price of the contract, which means that the market collapses. Such a situation is equivalent to the negative marginal price for the free market. It is quite clear that if the least price is implemented, the large volumes of goods cannot be sold in equilibrium provided the temperature is bounded. This does not mean the absence of deals. This means that the equilibrium cannot be achieved.

If $T = \frac{E}{N}$, the minimal price is equal to Q, and $V = \frac{E}{Q} = \frac{TN}{Q}$, so the part of goods equivalent to $V - \frac{TN}{Q}$ is impossible to realize.

A hasty observation may lead to the premature conclusion that the critical mass of the goods when the price is bounded from below will lie in a neighborhood of

$$V_{\text{crit}} = \frac{TN}{Q}. \tag{5.16}$$

But, as is clear from the above analysis, this is not the case: in reality the critical mass is considerably smaller. Namely, assuming that

$$P = \frac{TNQ}{E_0} \frac{1}{e^{\frac{QV_{\text{crit}}}{T}} - 1} > Q \tag{5.17}$$

or

$$\frac{TN}{E_0} > e^{\frac{QV_{\text{crit}}}{T}} - 1, \tag{5.18}$$

we obtain

$$V_{\text{crit}} = \frac{E_0}{Q} \ln\left(\frac{TN}{E_0} - 1\right). \tag{5.19}$$

As a result, instead of $V_{\text{crit}} = \frac{TN}{Q} = \frac{nE_0}{Q}$, deduced by the "naive" argument, V_{crit} is close to $\ln V$, i.e., is of several orders of magnitude smaller. Therefore, the market collapses much earlier than it seems at the first glance.

Coming back to the market with "lemons," this means that the ousting of good cars by the bad ones will happen much earlier than the naive a priori assumptions allow for.

Clearly, the asymmetrical information narrows the market's "zone of stability" to a very small size. Thus, our analysis justifies Hayek's conjecture that the market exists thanks to the culture of "honest business," and survives in the process of competition of business cultures.

The investigation above also points to another condition for existence of markets with high quality goods. The market operating with high quality goods has to be somehow separated from the market of low quality goods. Otherwise, with the influx of high quality commodities, the market becomes unstable.

The model outlined above shows that due to a limited supply of high quality goods, their market can coexist and interact with the market of "bad" goods, but only as the "market for elite," which means that outside its edges the market is stunted by the flaws in the "business code of behavior." An alternative is certification of goods.

We see that in a sense, our theory excludes "human component": the equilibrium theory in economics becomes parallel to that of statistical physics.

The approach we suggest is not supposed to be applied to technologies – an object of human activity.

It is also clear that hypothesis on equal probability of market states – the basis of any statistical theory – may be justified only if, following Hayek, we consider the human activity a principal factor subject to incomplete information on what's on at the market. Otherwise the hypothesis on equal probability of market state becomes absurd.

Human component is essential in one more aspect. The market requires abiding certain rules of contracts. In our theory we assume this as a given reality. This, however, is never so. Even if contracts are not restricted by "external" rules, **the mere necessity to abide the contracts makes the market into a social institute**.

The behavior of this institute, considered as a machine, in particular, its thermodynamic properties are of huge interest, but first such a machine – such an institute – must be created and being created should be maintained. Its collapse is a universal disaster.

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