SUMMARY OF RESULTS OF 1985 IRISH INVITATIONAL MATHEMATICS CONTEST

The Third Irish Invitational Mathematics Contest was held on Tuesday, March 19, 1985. Those who scored 80 or more in the IMMC were invited to take part in the INMC; returns were received on behalf of 29. Contestants had three hours to answer fifteen questions with integer solutions; partial credit was not given. Colm Morgan from Abbey Grammar, Newry, Co. Down, was our top scorer; Colm got eight questions correct, a very fine performance on what was a difficult test.

Here is a selection of the questions:

(3) Find c if a, b and c are positive integers which satisfy 
   \[ c = (a + bi)^3 - 1071, \] 
   where \( i^2 = -1 \).

(6) As shown in the figure on the right, \( \triangle ABC \) is divided into six smaller triangles by lines drawn from the vertices through a common interior point. The areas of four of these triangles are as indicated. Find the area of \( \triangle ABC \).

(7) Assume a, b, c and d are positive integers such that 
   \[ a^3 = b^3, \quad c^3 = d^3 \] 
   and \( c - a = 19 \). Determine \( d - b \).

(8) An ellipse has foci at \( (9,20) \) and \( (49,55) \) in the xy-plane and is tangent to the x-axis. What is the length of its major axis?

(This would have been beyond the scope of most Irish students. It is, however, a very nice question. Tom Laffey thinks that the principles behind the solution make a theorem that was missed by our predecessors. I myself incorporated the result into a course on Conics, which I gave to B.A. students this year.)

(12) Let A, B, C and D be vertices of a regular tetrahedron, each of whose edges measures 1 metre. A bug, starting from vertex A, observes the following rule: at each vertex it chooses one of the three edges meeting at that vertex, each edge being equally likely to be chosen, and crawls along that edge to the vertex at its opposite end. Let \( p = n/729 \) be the probability that the bug is at vertex A when it has crawled 7 metres. Find the value of \( n \).

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