PERSONAL ITEMS

Dr Ray Ryan of the Mathematics Department, UCC, is visiting the Department of Mathematics at Kent State University for the academic year 1985-'86.

Professor Robin Haute of the Mathematics Department, UCC, will be at the University of Iowa on sabbatical leave during the academic year 1985-'86.

Dr Gabrielle Kelly of the Statistics Department, UCC, will be at Columbia University on sabbatical leave during the academic year 1985-'86.

Dr Niall Ó Muinchadha of the Department of Experimental Physics, UCC, will spend the period September 1985-March 1986 on leave at the University of British Columbia.

Máiread Ó Seasúidh has been appointed to a temporary lectureship at the Mathematics Department, UCC, for the academic year 1985-'86. His research interests are in Operator Theory.

Mr Alastair Wood has been appointed to the Westinghouse Chair of Applied Mathematical Sciences at NIHE, Dublin.

A MATRIX JOKE

Robin Haute

1. If \( x = (x_{ij}) \in \mathbb{A}^{n \times n} \) is an \( nxn \) matrix with entries \( x_{ij} \) in a ring \( \mathbb{A} \) with identity 1, under what conditions does it have a two-sided inverse \( x^{-1} \in \mathbb{A}^{n \times n} \)? If the ring \( \mathbb{A} \) is commutative, then the answer is very nearly the same as for the real or the complex numbers:

\[
x \text{ invertible in } \mathbb{A}^{n \times n} \iff |x| \text{ invertible in } \mathbb{A}, \tag{1.1}
\]

where \( |x| \) denotes the determinant of \( x \), defined [5, Chapter 5] in any one of the usual ways. If the ring \( \mathbb{A} \) is not commutative then the formulae for the determinant become ambiguous, unless we restrict to matrices \( x = (x_{ij}) \) which are commutative, in the sense that their entries form a commutative set \( \{x_{ij}\} \). With this restriction implication (1.1) was demonstrated for 2x2 matrices of Hilbert space operators by Halmos [1, Problem 55], extended to nxn matrices of Banach algebra elements using the spectral mapping theorem [3, Example 2.4], and is now given in full generality by Halmos again [2, Problem 70]. In this note we will demonstrate that (1.1) holds separately for left and right inverses, at least for 2x2 matrices: the argument seems to depend on a joke.

2. Suppose that \( x = (x_{ij}) \) is a commutative \( nxn \) matrix over the ring \( \mathbb{A} \), with determinant \( |x| \in \mathbb{A} \), and cofactor \( x^{-} \in \mathbb{A}^{n \times n} \), in the sense of the usual 'adjugate' or 'classical adjoint' matrix of \( x \); then we recall Cramer's rule,

\[
x^{-}\times x = xx^{-} = |x|\mathbf{1}, \tag{2.1}
\]

and

\[
\mathbf{1}^{-} = \mathbf{1},
\]

where \( \mathbf{1} = (\delta_{ij}) \) is the identity matrix. If also \( y = (y_{ij}) \) is another commutative matrix, and if in addition the entries of