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Mechanical Criteria for Decomposition into Subdomains

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1 Introduction

Domain decomposition methods are widely used in several mechanical applications as, for example, non-linear elasticity or dynamic problems; we propose here, to extend their application to the study of heterogeneous structures. In the next section, we describe a methodology which uses homogenization techniques and subdomain decomposition methods to simulate behavior of composite material (which are strongly heterogeneous structures) and we shall see how it may conduct to good time savings on parallel computers. To this end, we will describe the difficulties encountered, which essentially consist in choosing a decomposition taking into account some mechanical criteria as the presence of heterogeneities. In the subsequent section, starting from a simple example concerning an elastic heterogeneous structure, we shall illustrate the influence of the decomposition, and specially the 'corner's problem' on the efficiency of the Schur complement method.

2 Review of Composite Materials and Homogenization Techniques

Composite materials are becoming more and more important in the construction of high-performances mechanical structures, as for example aerospace applications. Such applications necessarily require a good knowledge of the composite's material properties. The main difficulty in this area stems from the high level of heterogeneity encountered, making any numerical computation prohibitive if not impossible. A way to overcome these difficulties consists in using homogenization technique for periodic structures [Duv76, L$\acute{8}$4].

This technique first consists in considering two scales: the microscopic (connected to the composite basic cell) describes the composite's constituents and the macroscopic relates to the scale of the composite material studied. By a microscopic approach, this technique allows one to compute the equivalent homogeneous behavior for the composite. The computation of all moduli describing this homogeneous behavior

Ninth International Conference on Domain Decomposition Methods
Editor Petter E. Bjørstad, Magne S. Espedal and David E. Keyes

is carried out by the solution of problems to be solved at the microscopic level (called cellular problems), whose number and complexity depend on the composite's constituents. The main advantage of this theory is that the computation of the response of a homogeneous structure may be carried out without numerical overcosts. Let us note that this theory has led to the elaboration of many softwares and their efficiencies have been proved many times for several constituent's behaviors.

However the analyze [Dev92] of the CPU times required for this homogenization procedure reveals that 50 percent, is dedicated to the solution of the cellular problems. The first reason to this fact stems from the multiplicity of the cellular problems (for example in linear elasticity 6 cellular problems have to be solved, in order to obtain all moduli of the homogeneous behavior); the second reason consists in the presence of periodicity boundary conditions which affect the bandwidth of the FEM matrix. Thus, when concepters look for the optimal conception of composite materials, because they have to consider a lot of mechanical (moduli of the constituent's behavior)and geometric parameters (volume of inclusions, porosity part,...) to describe the microscopic level for quantifying their effects on the equivalent behavior moduli, it is necessary to multiply by a lot of parameters number the number of the cellular problems to be solved and it is clear that this conception step involves big difficulties when sequential computers are used.

Homogenization Process and Domain Decomposition Method.

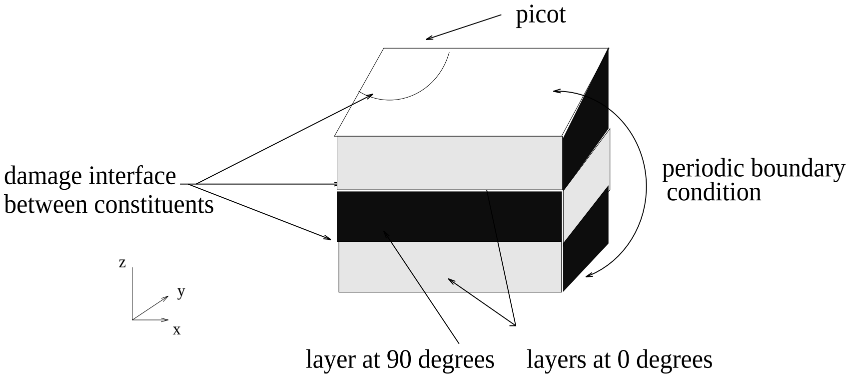
A way to overcome these difficulties, consists in developing [Sou96] a methodology which uses conjointly homogenization techniques and subdomain decomposition methods in order to take advantage in a large way of the parallel computers performance.

To illustrate this methodology, let us consider the case of a thermoelastic composite material, used at industrial level as coating for space engines. It is constituted by unidirectional carbon layers (0/90 degrees) held in contact by a third constituent (carbon) called 'picot'. For this geometry it is shown [L'H96] that the equivalent homogeneous thermoelastic behavior is orthotropic and can be fully computed by the solution of 6 elastic, 3 thermal and 1 coupled cellular problems. Moreover the analyze of stress concentration shows that damages located at the interfaces between constituents must be considered in order to obtain an accurate description of the composite reality. The example studied possess some geometric symmetries in the plane (0,x,y) enabling to carry out the solution of cellular problems only on the quarter of the basic cell (Figure 1). However periodicity boundary conditions between the lower and the upper faces of the quarter of the basic cell, remain. The parallel methodology developed for the computation of the composite damaged equivalent behavior consists in the cellular problems by use solving of a nonoverlapping subdomain decomposition method; we use, here, the Schur complement method [BW86]. To this way, we have to choose a subdomain decomposition for the quarter of the basic cell, accounting the periodicity boundary conditions and the discontinuities arising from damage interfaces.

Decompositions into Subdomains of the Basic Cell

- **Periodic boundary conditions in the decomposition.**

Figure 1 Quarter of the basic cell.



If the chosen decomposition is such that faces, where we have to satisfy periodic boundary conditions, belong to two different subdomains, it is then required to prescribe a particular link between these subdomains. Between each point of these faces we have to satisfy the **periodicity of the displacement** and the **antiperiodicity of the stress vector**. These relations being exactly the **same** than these to be satisfied at the interface of the decomposition, the main idea for treating these particular conditions (by a subdomain approach) consists in creating a 'fictive' interface, linking the d.o.f. belonging to faces concerned by periodic boundary conditions, and in adding it to the interface problem which will be classically solved by the conjugate gradient algorithm.

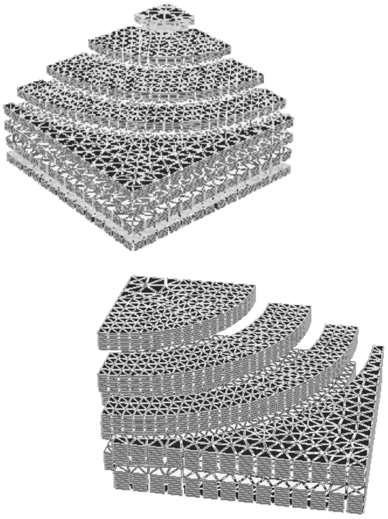
- **Damage interface in the decomposition.**

Two manners to take into account discontinuities in the meshes (modelizing the debonding interfaces) have been considered. They lead to two decompositions in respectively 20 and 8 subdomains (Figure 2). The first one consists in choosing the interfaces of the decomposition as those where damage occurs; when a d.o.f. comes in a damaged zone,(where no relations of continuity have to be satisfied), it is then picked out from the interface problem (decomposition in 20 subdomains). The second manner does not contain this association between damage and decomposition interfaces, and discontinuities arising from damage can be located inside subdomains (decomposition in 8 subdomains).

Results on the KSR Computer

We report (Figure 3) for each decomposition some results obtained during sequential process and parallel process (where the computing tasks related to each subdomain are carried out by each processor assigned). These results concern the solution of one cellular problem by use of the Schur complement method where the two 'Neumann' preconditioners have been considered ('Neumann-Neumann' [LeT94] and 'Neumann-coarse' [Man94]). They have been obtained on the KSR parallel machine of the INRIA institut. We dissociate in these curves the 'local operations' in the subdomains (computations of 'Dirichlet' and 'Neumann' problems) from the 'interface operations'

Figure 2 Decompositions in 20 et 8 subdomains of the quarter of the basic cell



(conjugate gradient operations). These results show that the decomposition in 20 subdomains is not satisfying because during the parallel process, most of time is consumed by the 'interface operations' in opposition to the decomposition in 8 subdomains for which the time saving may be increased by using more than one processor by subdomains. However, because we have chosen a damage located at the interface between the 'picot' (whose radius is small) and the layers, we have associated, for the decomposition in 20 subdomains, only one subdomain for the 'picot'. Thus we have spoiled the load balancing between processors. The decomposition in 8 subdomains does not present this phenomenon since it satisfies some mechanical criteria and a good size equilibrium between subdomains for a parallel application. With this example we raise the problem posed by the influence of the choice of the decomposition on times savings obtained on parallel process. This aspect can be more fully described thanks to the following example which concerns the study of the rate of convergence of domain decomposition methods when applied to heterogeneous structures where 'corners' exist.

3 Corner's Problem for Heterogeneous Structures

The use of the preconditioned conjugate gradient for the solution of the interface problem involves that the rate of convergence (and consequently the times savings) depends on the choice of the decomposition in subdomains ([SV95]), and recent researchs (see [FR94]) try to minimize this influence. To illustrate this aspect, let us consider the problem posed on the multilayered structure and describe in the Figure 4 (where σ is the stress tensor, $\epsilon(u)$) the linearized strain tensor, ν Poisson ratio and E the Young modulus). We study two material configurations: the first, is

Figure 3 Times repartition between operations of the Schur complement method

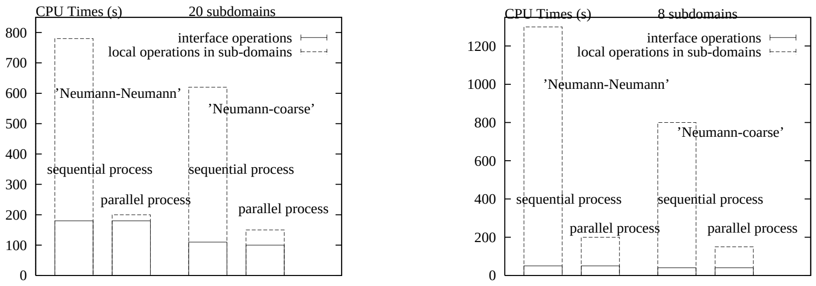
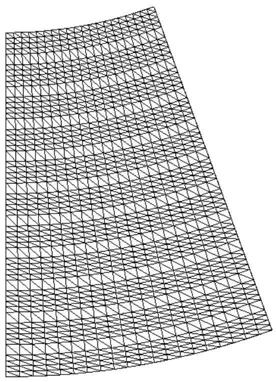


Figure 4 Problem posed on a multilayered structure.



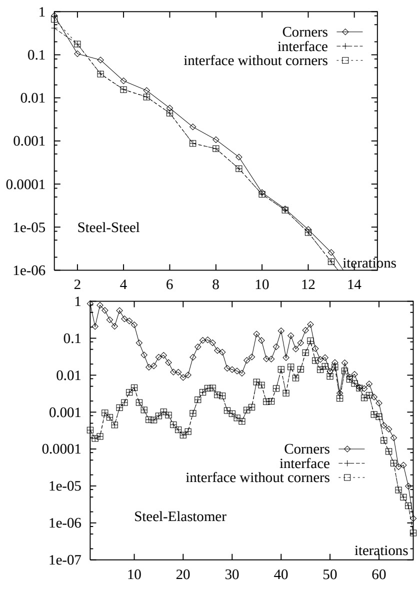
$div\sigma = 0$
$\epsilon_{ij}(u) = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{kk}\delta_{ij}$
$\epsilon(u) = \frac{1}{2}(\nabla(u) + \nabla^T(u))$
$u = 0$ on the lower face
$u = U$ on the upper face

homogeneous where all layer are made up of steel (called 'Steel-Steel'); the second is heterogeneous because the layers are alternatively made up of steel and elastomer (called 'Steel-Elastomer'). Four decompositions in 4, 6, 8 and 16 subdomains have been generated. They respect a good load balancing. Moreover some points of subdomain meshes belong to more than 2 subdomains (these points are called 'corners'). For each decomposition, we describe in Table 1.1 the rate of convergence (in iteration's number) of the Schur complement method used with the 'Neumann-coarse' preconditioner. Our purpose here is just to establish that in one case (homogeneous) we obtain a good efficiency of the 'Neumann-coarse' preconditioner as the subdomain number grows, whereas in the other case (heterogeneous) **with the same decomposition** we do not obtained it. To explain this phenomenon, we reporte in figure 1.5 the evolution of the conjugate gradient residual for each point of the interface (for the decomposition in 6 subdomains) and for the two material configurations studied. In these curves,

Table 1 Rate of convergence of the Schur complement method.

number of subdomains	number of corners	iterations with "Neumann-coarse" Steel-Steel — Steel-Elastomer
4	1	12 — 35
6	2	13 — 67
8	3	13 — 84
16	9	16 — 114

Figure 5 Residual of interface points, decomposition in 6 subdomains.



we dissociate in the global interface d.o.f(denoted by 'interface') the 'corners' d.o.f (denoted by 'corners') from the rest of d.o.f. (denoted by 'interface without corners'). It appears that in the heterogeneous case, the residual at the corner points is not decreasing like at the other interface points. We observe the same phenomenon for the other decompositions ([Sou96]). Thus the explanation of the deficiency of this preconditioner stems from the existence of corner points, which require (like in the case of the 'Two-level FETI Method' ([FM96])) the development of an adequate procedure for their treatments.

4 Conclusion

We have tried to pointed out with the examples studied, the problem encountered by the choice of decomposition in subdomains, when strongly heterogeneous structures are considered. Let us note that heterogeneities could be treated, however, by the coarsening operator [FR95], during the preconditioning step, but the decomposition step remains necessary and requires attention as we saw it with the example of the multilayered structure. However we have shown with the study on composite material that it is possible to use the interface relations to take into account some mechanical criteria as periodicity boundary conditions or the presence of damage interfaces.

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