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ON THE STRUCTURE OF AUTOMORPHISMS OF MANIFOLDS*

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Abstract. Thurston [16] proved that the group $\mathrm{Diff}^{\infty}(M)$ of a smooth manifold M is perfect, which implies the first homology group is trivial. If M has a geometric structure, then the first homology of the group of automorphisms of M preserving the geometric structure is not necessarily trivial. There are many results concerning this field. In this paper, we shall summarize the results of the first homology groups of automorphisms of manifolds with geometric structure.

Introduction

In this paper we shall report on the first homology group of the group of automorphisms of a manifold with geometric structure. Here the first homology group of a group is the quotient group of the group by its commutator subgroup. Let M be a connected closed smooth manifold. Let $\mathrm{Diff}^{\infty}(M)$ denote the group of C^{∞}-diffeomorphisms of M which are isotopic to the identity. Thurston [16] proved that $\mathrm{Diff}^{\infty}(M)$ is perfect which implies the first homology group is trivial. The result is related to the topology of the classifying space of foliations. There are many analogous results on the group of automorphisms of a manifold M which preserve a geometric structure on M such as volume structure, symplectic structure, submanifold structure, foliated structure, G-manifold structure. In those cases the first homology groups are not necessarily trivial. Then the calculation of the first homology is the next problem. The first homology

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