

## EXTREMALS OF THE GENERALIZED EULER-BERNOULLI ENERGY AND APPLICATIONS\*

ÓSCAR J. GARAY

*Departamento de Matemáticas, Facultad de Ciencia y Tecnología  
 Universidad del País Vasco, Aptdo 644, 48080 Bilbao, Spain*

**Abstract.** In this survey we describe a general method to deal with the variational problem associated to the generalized elastic curves, paying special attention to closed critical curves in real space forms due to its special geometric significance. We illustrate the method by studying particular choices of this energy in some more detail. Finally, we will review also some interesting applications of generalized elasticae to other higher dimensional variational problems in Physics, Biophysics and the Theory of Submanifolds.

### CONTENTS

1. Introduction .....	57
2. Preliminaries .....	58
2.1. A Useful Tool I: Hopf Cylinders .....	59
2.2. A Useful Tool II: Lancret's Curves .....	59
3. Order one Functionals .....	60
4. Higher Order Functionals .....	65
5. Natural Choices of the Energies .....	70
5.1. Classical Elasticae in $\mathbb{S}^3(\mathbf{1})$ .....	71
5.2. Elastic Curves Circular at Rest .....	74
6. Some Applications .....	77
6.1. Models of Relativistic Particles .....	77
6.2. From Membranes in Biophysics to Worldsheets in String Theory ...	79
6.3. Chen-Willmore Submanifolds .....	82

\*Reprinted from JGSP **12** (2008) 27–61.