

ON SURFACES OF LOW GENUS WHOSE TWISTOR LIFTS ARE HARMONIC SECTIONS*

KAZUYUKI HASEGAWA

Institute of Human and Social Sciences, Faculty of Teacher Education, Kanazawa University, Kakuma-machi, Kanazawa, Ishikawa, 920-1192, Japan

Abstract. We determine surfaces of genus zero in four-dimensional hyperkähler manifolds whose twistor lifts are harmonic sections. As corollary, we prove that a surface of genus zero in four-dimensional Euclidean space is twistor holomorphic if its twistor lift is harmonic section.

1. Introduction

The twistor lifts play an important role and have been studied for oriented surfaces in oriented four-dimensional Riemannian manifolds by many researchers. For examples, in [2], it is proved that any oriented surface admits a conformal, superminimal immersion into the four-dimensional sphere. The twistor space is endowed with an almost complex structure, which is integrable when the base manifold is self-dual (see [1]). Surfaces with holomorphic twistor lifts are called twistor holomorphic surfaces (see [6]). The surfaces whose twistor lifts are harmonic sections have been studied in [8] and [9]. If the ambient spaces are self-dual Einstein, the twistor lifts of twistor holomorphic surfaces are harmonic sections. Note that recently surfaces whose twistor lifts are harmonic sections have been studied from the view point of the integrable systems in [3] and [11].

In this paper, we announce results for surfaces of genus zero in four-dimensional hyperkähler manifolds whose twistor lifts are harmonic sections. If certain conditions for the curvature tensors are satisfied and the twistor lift is a harmonic section, then the mean curvature vector field is a holomorphic section of the normal bundle with respect to the Koszul-Malgarange holomorphic structure (see [3], [8] and [9]). In general, holomorphic sections are not parallel. It is well-known that a compact constant mean curvature surface of genus zero immersed in \mathbb{R}^3 is totally umbilic,

*Reprinted from JGSP **17** (2010) 35–43.