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WEAK FORM OF HOLZAPFEL'S CONJECTURE*

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Abstract. Let $\mathbb{B} \subset \mathbb{C}^2$ be the unit ball and Γ be a lattice of $\mathrm{SU}(2,1)$. Bearing in mind that all compact Riemann surfaces are discrete quotients of the unit disc $\Delta \subset \mathbb{C}$, Holzapfel conjectures that the discrete ball quotients $\mathbb{B}/\Gamma$ and their compactifications are widely spread among the smooth projective surfaces. There are known ball quotients $\mathbb{B}/\Gamma$ of general type, as well as rational, abelian, K3 and elliptic ones. The present note constructs three non-compact ball quotients, which are birational, respectively, to a hyperelliptic, Enriques or a ruled surface with an elliptic base. As a result, we establish that the ball quotient surfaces have representatives in any of the eight Enriques classification classes of smooth projective surfaces.

1. Introduction

In his monograph [4] Rolf-Peter Holzapfel states as a working hypothesis or a philosophy that "... up to birational equivalence and compactifications, all complex algebraic surfaces are ball quotients." By a complex algebraic surface is meant a smooth projective surface over $\mathbb{C}$. These have smooth minimal models, which are classified by Enriques in eight types - rational, ruled of genus ≥ 1, abelian, hyperelliptic, K3, Enriques, elliptic and of general type. The compact torsion free ball quotients $\mathbb{B}/\Gamma$ are smooth minimal surfaces of general type. Ishida [10], Keum [11,12] and Dzambic [1] obtain elliptic surfaces, which are minimal resolutions of the isolated cyclic quotient singularities of compact ball quotients. Hirzebruch [2] and then Holzapfel [3], [7], [9] have constructed torsion free ball quotient compactifications with abelian minimal models. In [9] Holzapfel provides a ball quotient compactification, which is birational to the Kummer surface of an abelian surface,

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