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# ARITHMETIC PROPORTIONAL ELLIPTIC CONFIGURATIONS WITH COMPARATIVELY LARGE NUMBER OF IRREDUCIBLE COMPONENTS

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**Abstract.** Let $T$ be an arithmetic proportional elliptic configuration on a bi-elliptic surface $A_{\sqrt{-d}}$ with complex multiplication by an imaginary quadratic number field $\mathbb{Q}(\sqrt{-d})$. The present note establishes that if $T$ has $s$ singular points and

$$4s-5 \leq h \leq 4s$$

irreducible smooth elliptic components, then $d=3$ and $T$ is $\mathrm{Aut}(A_{\sqrt{-3}})$-equivalent to Hirzebruch's example $T^{(1,4)}_{\sqrt{-3}}$ with a unique singular point and 4 irreducible components.

In [3], it was announced "as a working hypothesis or a philosophy" that … "*up to birational equivalence and compactifications, all complex algebraic surfaces are ball quotients.*" This was proven it for the abelian surfaces. In order to formulate it precisely, one needs the following

**Definition 1** (Holzapfel [5]). *A reduced effective divisor $T$ on an abelian surface $A$ is called an intersecting elliptic configuration if all the irreducible components $T_i$ of $T$ are smooth elliptic curves with $s_i := \mathrm{card}(T_i \cap T^{\mathrm{sing}}) \geq 1$, and all the non-void intersections $T_i \cap T_j \neq \emptyset$, $i \neq j$ are transversal.*

**Definition 2** (Holzapfel [5]). *An intersecting elliptic configuration $T = T_1 + \cdots + T_h$ on an abelian surface $S$ is proportional if*

$$s_1 + \cdots + s_h = 4s$$

*for $s := \mathrm{card}(T^{\mathrm{sing}})$, $s_i := \mathrm{card}(T_i \cap T^{\mathrm{sing}})$.*

**Theorem 1** (Holzapfel [5]). *An abelian surface $A$ is the minimal model of the toroidal compactification $(\mathbb{B}/\Gamma)'$ of a neat ball quotient $\mathbb{B}/\Gamma$ if and only if $A = E \times E$ is bi-elliptic and there exists a proportional elliptic configuration $T \subset A$.*