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FLUX CONJECTURE ON SYMPLECTIC SUBMANIFOLDS

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Abstract. Let (M,ω) be a closed symplectic $2n$-dimensional manifold. According to the well-known result by Donaldson [5] there exist $2m$-dimensional symplectic submanifolds (V^{2m},ω) of (M,ω), $1 \leq m \leq n-1$, with $(m-1)$-equivalent inclusions. In this paper, we have found a relation between the flux group and the kernel of the Lefschetz map. We have present also some properties of the flux groups for all symplectic $2m$-submanifolds (V^{2m},ω) where $2 \leq m \leq n-1$.

1. Introduction

Let (M,ω) be a compact symplectic manifold and $\mathrm{Symp}_0(M)$ denote the identity component of the symplectomorphism group $\mathrm{Symp}(M)$ of (M,ω). Recall that the flux homomorphism

$$F_\omega : \pi_1(\mathrm{Symp}_0(M)) \to H^1(M,\mathbb{R})$$

can be defined as follows. For an element $\phi \in \pi_1(\mathrm{Symp}_0(M))$ and any homology class $\alpha \in H_1(M,\mathbb{R})$ set

$$(F_\omega(\phi),\alpha) = (\omega, \phi_t\alpha)$$

where $\phi_t\alpha$ denotes the trace of a loop α under the isotopy $\{\phi_t\}$ representing ϕ and $(\cdot,\cdot)$ is the natural pairing. It is well known that ϕ is represented by a Hamiltonian loop if and only if $F_\omega(\phi)=0$. Define the flux group Γ_M of M by the image of the flux homomorphism, i.e.,

$$\Gamma_M = \mathrm{im}\{F_\omega : \pi_1(\mathrm{Symp}_0(M)) \to H^1(M,\mathbb{R})\} \subset H^1(M,\mathbb{R}).$$

The importance of this notion is due to the fact that the Hamiltonian diffeomorphism group $\mathrm{Ham}(M)$ is closed in $\mathrm{Symp}_0(M)$ if and only if Γ_M is a discrete subgroup of $H^1(M,\mathbb{R})$. The statement that Γ_M is discrete is known as the flux conjecture. Then we obtain a relation between the flux group Γ_M of M and