Eighth International Conference on
Geometry, Integrability and Quantization
June 9–14, 2006, Varna, Bulgaria
Ivaïlo M. Mladenov and Manuel de León, Editors
SOFTEX, Sofia 2007, pp 121–126

PROJECTING ON POLYNOMIAL DIRAC SPINORS

NICOLAE ANGHEL

Department of Mathematics, University of North Texas, Denton, TX 76203, USA

Abstract. In this note we adapt Axler and Ramey's method of constructing the harmonic part of a homogeneous polynomial to the Fischer decomposition associated to Dirac operators acting on polynomial spinors. The result yields a constructive solution to a Dirichlet-like problem with polynomial boundary data.

It is well-known [3] that any homogeneous real or complex polynomial p_k of degree $k = 0, 1, 2, \dots$ in $n \geq 2$ real variables $x = (x_1, x_2, \dots, x_n)$ admits an unique decomposition

$$p_k(x) = h_k(x) + |x|^2 p_{k-2}(x) \tag{1}$$

where h_k is a homogeneous harmonic polynomial of degree k, p_{k-2} is a homogeneous polynomial of degree $k-2$, and, as usual, $|x| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$.

In [1] Axler and Ramey presented an elegant, elementary way of constructing h_k from p_k, which involves only differentiation. In essence, for $k > 0$

$$h_k(x) = \begin{cases} c_k^{-1} |x|^{2k} p_k(D)(\log |x|), & \text{if } n = 2 \\ c_k^{-1} |x|^{n-2+2k} p_k(D)(|x|^{2-n}), & \text{if } n > 2 \end{cases} \tag{2}$$

where

$$c_k = \begin{cases} (-2)^{k-1}(k-1)!, & \text{if } n = 2 \\ \prod_{j=0}^{k-1}(2 - n - 2j), & \text{if } n > 2 \end{cases} \tag{3}$$

and where $p_k(D)$ is the associated partial differential operator acting on smooth functions defined on open subsets of $\mathbb{R}^n$ obtained by replacing a typical monomial $x_1^{\alpha_1} x_2^{\alpha_2} \dots x_n^{\alpha_n}$, $\alpha_1 + \alpha_2 + \cdots + \alpha_n = k$, of p_k by $\dfrac{\partial^k}{\partial x_1^{\alpha_1} \partial x_2^{\alpha_2} \dots \partial x_n^{\alpha_n}}$.

As a by-product they obtained a speedy solution to the Dirichlet problem on the unit ball of $\mathbb{R}^n$ with polynomial boundary data which eliminates the use of the impractical Poisson integral.