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ALGEBRAS WITH POLYNOMIAL IDENTITIES AND BERGMAN POLYNOMIALS

TSETSKA G. RASHKOVA

Department of Algebra and Geometry, A. Kanchev University of Rousse
7017 Rousse, Bulgaria

Abstract. This paper is an introduction to the theory of algebras with polynomial identities. It stresses on matrix algebras and polynomial identities for them. The notion of Bergman polynomials is introduced. Such types of polynomials are investigated being identities for algebras with symplectic involution. In the Lie case more information is given for Bergman polynomials as Lie identities for the considered algebras.

1. Algebras with Polynomial Identities

We fix a countably infinite set $X = \{x_1, x_2, \dots\}$ and consider a field K of characteristic zero. We work in the algebra $K\langle X\rangle$ which has a basis the set of all words

$$x_{i_1} \dots x_{i_k}, \qquad x_{i_j} \in X$$

and multiplication defined by

$$(x_{i_1} \dots x_{i_m})(x_{j_1} \dots x_{j_n}) = x_{i_1} \dots x_{i_m} x_{j_1} \dots x_{j_n}.$$

Definition 1. i) *Let $f = f(x_1, \dots, x_n) \in K\langle X\rangle$ and let R be an associative algebra. We say that $f = 0$ is a **polynomial identity** for R if*

$$f(r_1, \dots, r_n) = 0, \qquad r_1, \dots, r_n \in R.$$

ii) *If the associative algebra R satisfies a non-trivial polynomial identity f (i.e., f is a nonzero element of $K\langle X\rangle$), we call it PI-algebra.*

It could be shown that $f \in K\langle X\rangle$ is a polynomial identity for R if and only if f is in the kernel of all homomorphisms $K\langle X\rangle \to R$. We give some examples: